A SIMPLE LANGUAGE AND PLOTTING SYSTEM TO TEACH DESCRIPTIVE GEOMETRY

P.J. ZSOMBOR-MURRAY

DATAC COMPUTER LABORATORY
DEPARTMENT OF MECHANICAL ENGINEERING
McGILL UNIVERSITY

ABSTRACT

A graphics language, currently consisting of a 3-word vocabulary with modifying suffixes, is used to aid in the teaching of descriptive geometry. This language is used to describe, symbolically, the steps required to set up and solve simple problems of the class which involve construction of successive auxiliary views. Furthermore, a primitive interpreter translates the problem description into a formally formatted and labelled output on a digital plotter. Errors in logic and syntax are diagnosed at run time. The structure of the interpretive program is described. Sample exercises and proposed extensions of the language are discussed.

UN LANGAGE SIMPLE AVEC TRACE AUTOMATIQUE POUR L'ENSEIGNEMENT DE LA GEOMETRIE DESCRIPTIF

P.J. ZSOMBER-MURRAY

LABORATOIRE D'INFORMATIQUE DATAC
DEPARTEMENT GENIE MECANIQUE
UNIVERSITE McGILL

ABREGE

On emploie pour enseigner la géométrie descriptif un langage, qui se compose, présentement, d'un vocabulaire de 3 mots, modifiés par des suffixes. Ce langage décrit les étapes requises lors de la définition et de la résolution des problèmes simples. Le système comprend la construction des projections auxiliaires. Enfin, les instructions symboliques produisent, via le programme interprétif, un dessin, en forme conventionelle et étiquetée, sur une table à tracer automatique. Le programme décèle les erreurs logiques et syntactiques. La structure du programme, des exemples et des améliorations possible sont décrits.

A Simple Language and Plotting System to Teach Descriptive Geometry

Much of computer graphics has been preoccupied with constructing 2-dimensional representations of 3-dimensional objects so that these appear as if viewed in reality by a human observer. Hiding and perspective illusions have been produced. These, not to mention animation, incur large computational overhead. Justification of this exists only if the graphics is intended to aid in evaluating aesthetic merit of, say, an architectural design or if it is intended to train, exercise or test a sight dependant skill such as in a flight simulator. One tends to forget, however, that the quantitative usefulness of realistic images is limited a-priori because they are acquired via a pair of small and closely spaced eyes.

On the other hand, orthographic projections are used in mechanical design, almost exclusively, so that all lines parallel to the drawing plane appear in uniform scale. A line's image is shortened only by the cosine of the angle that the line makes with the plane. In this way all coordinate information is available in any conjugate pair of orthographic views. Each view is a 2-dimensional projection and the pair has a common, parallel coordinate. Auxiliary views, required to bring any arbitrary set of parallel planes into uniform proportion (i.e. true-view), are generated by first selecting the direction, in an existing view, which is to become the common, parallel coordinate in the subsequent auxiliary view. It should be noted that the directions perpendicular to the common coordinate, in each member of a conjugate view pair, are, although they appear to be parallel, in fact mutually perpendicular. This, then, is the ultimate perspective. Such a viewing system is owned by the being in Fig. 1 whose pair of eyes, on a spherical surface of infinite radius, observe the 3-dimensional universe along perpendicular lines of sight. To obtain successive auxiliary views, each eye is constrained to move, alternately, along the great circular trajectory in a plane perpendicular to the current, stationary line of sight. By successive, leap-frogging rotations either eye can view from any direction.

The advantages of orthographic projection as a design tool for 3-dimensional systems are that:-

1. All distances parallel to any projection plane are of uniform scale,
2. 3-dimensional data are reduced to pairs of 2-dimensional projections each member of which, when taken separately, is relatively easy to comprehend and
3. A simple transformation rule can produce any desired plane projection from the data contained in any conjugate view pair.

There are disadvantages as well:-

1. We are not equipped by natural experience to view and sensibly integrate conjugate, orthographic view pairs and
2. The clutter of overlapping lines belonging to separate, possibly nonconjugate, projections further confuses our interpretation of such data.

It would seem that these difficulties are part of the price which must be paid to effectively design in 3 dimensions. Engineering students must therefore be taught, via the medium of descriptive geometry, to construct and read designs composed of pairs of orthographic views. This is not easy because, in modern engineering curricula, training in the purely mechanical skills of drawing has been all but eliminated in favour of an ever increasing number of important analytical disciplines. The result of this is the absence of a large, trained clientelle to exploit existing computer aided designing systems and to generate demand for their evolution. It is felt that mechanical design in Canadian industry has become relegated chiefly to draftsmen.

A noteable exception to this trend in undergraduate teaching is the current interest in languages and systems, such as APT, to program and control manufacturing processes. Clearly, languages and systems to aid the designer are at least of equal import. It would seem that the potential for improvement of product cost and performance is as dependant upon the design as it is upon good production methods. Moreover, line drawing should be easier to automate than surface generation. It is doubtful, however that many would quarrel with the observation that there is substantially greater industrial acceptance of, say, numerically controlled machine tools than there is of computer aided designing systems. Why is this?

In order to ensure the utility of any computer graphics one must adhere closely to existing convention. Traditional representation is however frequently ignored. Often, the system designer (not to be confused with the beleaguered mechanical designer) prefers to impose his own ad-hoc conventions based on programming convenience or even some arbitrary whim of momentary expedience. The essential aspect of translating mental conception into a drawing is the gradual, <u>simultaneous</u> evolution of a pair of partial principal views and also the generation from these, of auxiliary views as required. At no time can the designer visualize the entire detail of his creation. He must however formulate all details, albeit piecemeal. The drawing is a data store and retrieval system with which he continually interacts during the design process. It is doubtful whether computer aided drafting systems are used in this manner. Those which have the capacity to accomodate this mode of operation are elaborate and expensive. Others, less expensive, are merely output systems with limited, if any, capacity for interactive editing. Possibly these inexpensive systems might be made more useful through the development of suitable languages for specific purposes.

Consider then the vocabulary, syntax and constraints of the simplest language which may be used to describe and command the construction of orthographic view pairs. Its convention in symbols and graphics resemble those of descriptive geometry because they are concise and unambiguous and because a descriptive geometry problem contains many of the aspects of mechanical drawing without requiring a very large data base. For further simplification, only those constructions composed of points, lines and planes have been considered.

<u>Language Resumé</u>:-

1. Begin
2. Define a point
3. Define a line
4. Define an auxiliary view

5. Terminate

Interpreter Resumé:-

1. Initialize data base and plot VH-line
2. Read instruction
3. Interpret instruction
4. Maintain and update data base
5. Plot graphics and symbols as per instruction
6. Terminate

Language Functions:-

1. Begin
   a) This is implicit in turning on the interpreter program, DGDØ
   b) This function has no language instruction but is part of the JCL (job control language) of the operating system
   c) The interpreter is started by the following statement

sample card image

```
JØB,DGDØ
1                        ... card column number
```

2. Define a point
   a) This is commanded by the instruction, PT
   b) A point label is specified by the first operand, (p)
   c) 3 coordinates are specified as the following operands in 3(F6.n,1X) FØRTRAN format as (sxx.xx,syy.yy,szz.zz)
   d) The suppression of a point and its label is commanded by placing an S after the point label

sample card images

```
PT(A)(ƀƀ4.00,-ƀ6.00,ƀƀ3.5ƀ)... optional comment
PT(RS(+11.00,ƀ-2.25,-ƀ9.ƀƀ)... optional comment
1  4  7      14      21     28
```

3. Define a line segment
   a) This is commanded by the instruction, LN
   b) Two point labels, the extent of the segment, are specified in the operand, (qr)
   c) Anything other than spaces placed in the following 3 columns will suppress the principal projections of the commanded line segment

sample card image

```
LN(BC)ƀƀƀ... optional comment
1  4  7  10
```

4. Define an auxiliary view
   a) This is commanded by the instruction, A
   b) The principal view from which the auxiliary conjugate se-sequence began is specified as a 1st operand, H or V
   c) The number of conjugate pairs preceding the commanded auxiliary view is specified as 1, 2 or 3 after AH or AV. Note that a 1st auxiliary view is preceded by only the principal pair while a 3rd auxiliary is preceded by a principal V-H or H-V pair, a principal-1st and a 1st-2nd pair
   d) The direction of the common parallel coordinate of the conjugate pair created by the current instruction is specified by $\underline{=}$ or $\underline{+}$ as the 3rd operand. Note that $\underline{=}$ signifies parallel

to and $\perp$ means perpendicular to...

e) ...a key line, already defined in two PT instructions, and specified as the 4th operand, i.e. qr. Note that it is unnecessary to define the line segment as LN(qr) in order to generate an auxiliary view

f) Anything other than spaces placed in card columns 7 through 9 inclusive will suppress the commanded auxiliary view while enabling the specification of its logical successor

sample card images

```
AH1+ABb(SUPPRESSED VIEW)

AH2=PQbbb... optional comment
1 3 5 7  10
```

5. Terminate
   a) This is commanded by 3 consecutive spaces in the 1st 3 columns of an instruction card

sample card image

```
bbb... optional comment
1  4
```

Interpreter Functions:- (Fig. 2, block diagram)

1. Initialize data base and plot VH-line
   a) The interpreter maintains tables, as shown in Fig. 3, to hold the following data:-
      i. Point labels, in a vector, (P), of 20 locations, each containing an ASCII character in A1 format,
      ii. Line segment identifiers, in a vector, (L), of 40 locations, each containing 2 ASCII characters in A2 format,
      iii. Principal coordinates of up to 20 points in a 3x20 matrix, (X,Y,Z)
      iv. Auxiliary coordinates of up to 20 points, 3 coordinates per point, in 6 auxiliary views; 3 auxiliary views belonging to either conjugate view sequence beginning with the H or V principal projection, respectively. These are contained in a 6x3x20 matrix, $((X_1,W_1,W_1'),(X_2,W_2,W_2'), \ldots ,(X_6,W_6,W_6'))$, and
      v. The positive (anticlockwise from the VH-line) angle between the new, rotated auxiliary x-axis or key line direction and the plane edge separating the preceding conjugate view pair. This is stored in the 6 locations of vector (A).
   b) At initialization, all 476 table locations are set empty.
   c) A VH-line, 5.5 in. long, is drawn. This represents, alternately, the edge of the V-principal projection plane viewed in the H-plane and the edge of the H-plane viewed in the V-projection. This is the horizontal line, in Fig. 4, with the letters VH drawn over the origin

2. Read instruction
   a) Prior to interpretation, the instruction card image is stored in a 5 location buffer
   b) The 1st 2 locations contain the 1st 6 characters, punched in the 1st 6 columns of the instruction card, in 2A3 format
   c) The next 3 words store information contained in the columns 7-26 inclusive, as 3 numbers in 3(F6.n,1X) format

d) This information is retained until overwritten by the next card read

3. Interpret instruction
   a) The 1st 2 characters stored in the card image buffer are scanned to determine if they contain:-
      i. PT or
      ii. LN or
      iii. AH or AV or
      iv. ~~bb~~ or
      v. Anything else, which is undefined, hence implicitly illegal
   b) Illegal instructions are ignored but produce error messages. The interpreter waits 20 sec. for a signal which, if received, will cause the next read instruction. The task is aborted if no signal occurs
   c) A legal instruction activates the appropriate processor

4. Maintain and update data base
   a) A PT(p)(... instruction is illegal if $X_p^2+Y_p^2+Z_p^2 > 5.5^2$
   b) The point label, p, is stored in the 1st empty location encountered in vector (P)
   c) In PT(p)(X_p....,Y_p....,Z_p....), X_p, Y_p and Z_p are stored in the first empty row found in matrix (X,Y,Z)
   d) An LN(pq) instruction is considered illegal if p or q is not found in vector (P)
   e) The end points, pq, of the specified line segment are stored in the 1st empty location found in vector (L)
   f) An AH or AV instruction is illegal if:-
      i. In AHn, n is anything other than 1, 2 or 3,
      ii. AH2 is encountered and table columns (X_1,W_1,W_1') are empty,
      iii. AH3 is read and table columns (X_3,W_3,W_3') are empty,
      iv. AV2 is read and table columns (X_2,W_2,W_2') are empty,
      v. AV3 is read and table columns (X_4,W_4,W_4') are empty,
      vi. In AHnd, d is anything other than $\underline{=}$ or $\underline{+}$,
      vii. In AHndpq, either p or q is not found in vector (P).
   g) The counterclockwise angle, between the key line (pq in AHndpq) projection and the direction of the common coordinate in the conjugate view pair immediately before (in view, not instruction sequence) the view commanded by AHndpq, is put into vector (A). Refering to Fig. 5 and noting that i is parallel to pq and perpendicular to pr (points q and r not shown), A_i is computed as:-

$$A_i = \tan^{-1}[(W_{q_{i-1}}-W_{p_{i-1}})/(X_{q_{i-1}}-X_{p_{i-1}})]$$

   if d is $\underline{=}$ and as:-

$$A_i = \tan^{-1}[(X_{p_{i-1}}-X_{q_{i-1}})/(W_{q_{i-1}}-W_{p_{i-1}})]$$

   if d is $\underline{+}$
   h) $X_i(k)$, $W_i(k)$ and $W_i'(k)$ are computed and stored in the k'th row of (X,W,W'). These relations are seen to be:-

$$X_i = [\cos(A_i)]\sqrt{X_{i-1}^2 - W_{i-1}^2},$$

$$W_i = W_{i-1}' \quad \text{and}$$

$$W_i' = \sin\left[\tan^{-1}\left[\left(\frac{W_{i-1}}{X_{i-1}} - A_i\right)\right]\right]\sqrt{X_{i-1}^{\,2} + W_{i-1}^{\,2}}$$

Note, in Fig. 5, that W_{i-1}' bears the same relationship to $\underline{i}$ as W_i' will bear to the next common coordinate direction, $\underline{i+1}$, which is not shown

i) Only 3 conjugate auxiliary view pairs proceeding, respectively, from the H- and V-principal projections can exist. This is deemed to be enough since a 3rd auxiliary view, corresponding to 3 successive rotations of the perpendicular eyes, can produce a projection plane parallel to any arbitrary plane

j) Note, however, that a new auxiliary sequence can be started at any level. E.g., a 2nd AH3dpq instruction may be submitted after a second AH1dpq instruction. The new 1st auxiliary projection, proceeding from the H-projection, will replace the original data in (X_1,W_1,W_1') and in (A_1). Similarly, the 2nd AH3dpq command will replace data previously in (X_5,W_5,W_5') but the new 3rd auxiliary will be computed from original data in (X_3,W_3,W_3') and in (A_3) since no new AH2dpq has been presented to overwrite the existant 2nd auxiliary view proceeding from H

5. Plot graphics and symbols as per instruction
   a) If the suppression option is invoked in any instruction, the interpreter bypasses the plot processor
   b) The PT... instruction causes the principal projections of the point to be marked by unequal crosses as shown in Fig. 4
   c) The point label PT(A)... is placed beside the point mark and is appropriately subscripted
   d) If 2 points have the same coordinates in any projection, the labels are displaced as is the case of A_4B_4 in Fig. 4
   e) The LN(AB) instruction causes the principal projections of A and B to be joined
   f) The A... instruction causes an edge view of the commanded auxiliary projection plane to be drawn as a centre line in the appropriate direction, at an angle, A_i, to the preceding common coordinate direction in the auxiliary view sequence, 5.5 in. long, beginning at the point (0,0,0) and labeled at the other end with 1, 3 or 5 if it belongs to an AH... sequence and is, respectively, a 1st, 2nd or 3rd auxiliary plane. Similarly, the label will be 2, 4 or 6 for an AV... sequence
   g) Auxiliary points are marked with a small circle
   h) All points and line segments which appear in principal views are reproduced in auxiliaries

6. Terminate
   a) The interpreter program relinquishes control to the operating system ECP (executive control program) which resumes reading cards, interpreting these as JCL instructions

Example Problems

Even with the primitive system which has been described, most of the fundamental problems of descriptive geometry can be assigned and solved. To name but a few, in order of increasing complexity:-

1. Obtaining the true length of an oblique line and the true angle at which it intersects a principal plane,
2. Obtaining the end view of a line, hence the shortest distance between any pair of oblique lines,
3. Obtaining the dihedral between a pair of intersecting, oblique planes,
4. Obtaining a view of the true area of an oblique plane and
5. Obtaining the angle of intersection between a line and a plane.

Solution Procedures

Problems 1., 2. and 3. can be illustrated by Fig. 4:-

1. A_2B_2 is in true length since the 1st auxiliary plane, 2, is parallel to A_VB_V. Since 2 represents the edge of the V-plane, its angle of intersection with A_2B_2 is in true view.
2. By obtaining the end view of AB, viz. A_4B_4, any other line, CD, (not shown) would be separated from AB by the closest distance described by a perpendicular line from the point A_4B_4 to the line projection C_4D_4.
3. Adding points C and D to Fig. 4 such that ABC represents one plane and ABD the other then AB is common to both. The angle C_4-A_4B_4-D_4 is the required dihedral as shown in Fig. 6.
4. Consider the plane ABC in 3. above. Obtain a 3rd auxiliary, 6, parallel to A_4B_4-C_4. $A_6B_6C_6$ is the true area as shown in Fig. 7.
5. Consider the plane PQR and a line EF. Suppose PQR is constructed so that $Z_P = Z_Q$, i.e. PQ is a horizontal line, hence P_HQ_H is in true view. Obtain a 1st auxiliary, P_1Q_1, end view with the edge $P_1Q_1R_1$. The true view $P_3Q_3R_3$ is obtained as per 4. above. A 3rd auxiliary parallel to E_3F_3 will show $P_5Q_5R_5$ in edge again with E_5F_5 now in true view. E_5F_5-$P_5Q_5R_5$ is the required angle. This is illustrated in Fig. 8.

Note the instruction sequences tabulated within Figs. 4, 6, 7 and 8

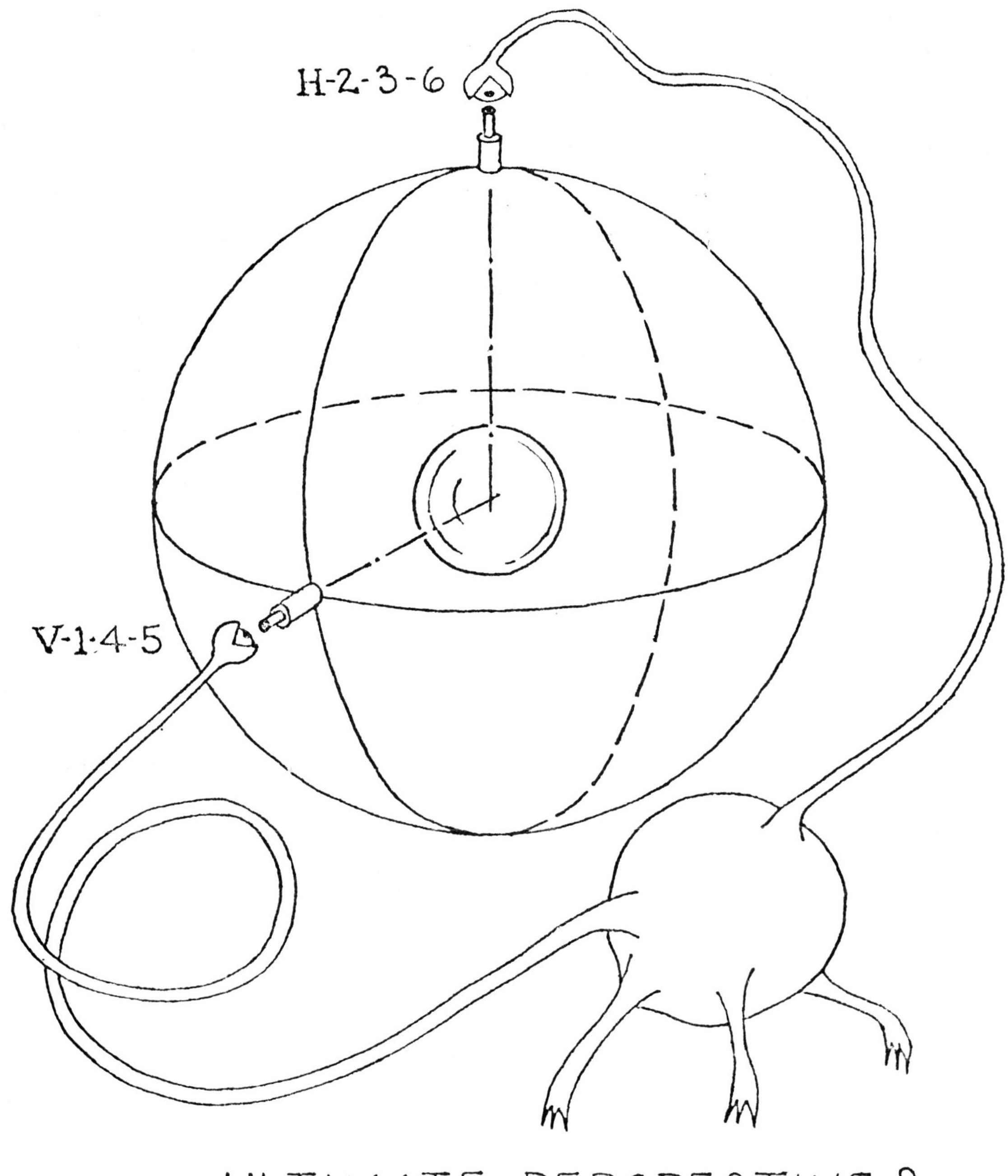
Proposed Extensions to the Language

It is felt that the 5 examples above demonstrate that, in spite of extreme simplicity, this language has the power if not the storage capacity to treat most problems involving plane figures. Development plans therefore include the specification of curved surface projections including conics, spheroids and their intersections. It is hoped to maintain the existing concise command structure in new instructions.

Vital Statistics

The program, DGDØ, including storage and all routines runs unsegmented in just over 4000 words (24-bit) in a GE/PAC 4020 under RTMØS with FTS04. Input is via cardreader on the background nonspooling batch stream. Except for error messages, output is on an online CALCØMP 565 digital plotter. The program has been used sparingly in the engineering graphics courses at McGill. One compulsory problem is assigned and few students are predisposed to use DGDØ except under compulsion. It would be safe to say that it is unloved, save by its author.

Fig. 1



ULTIMATE PERSPECTIVE?

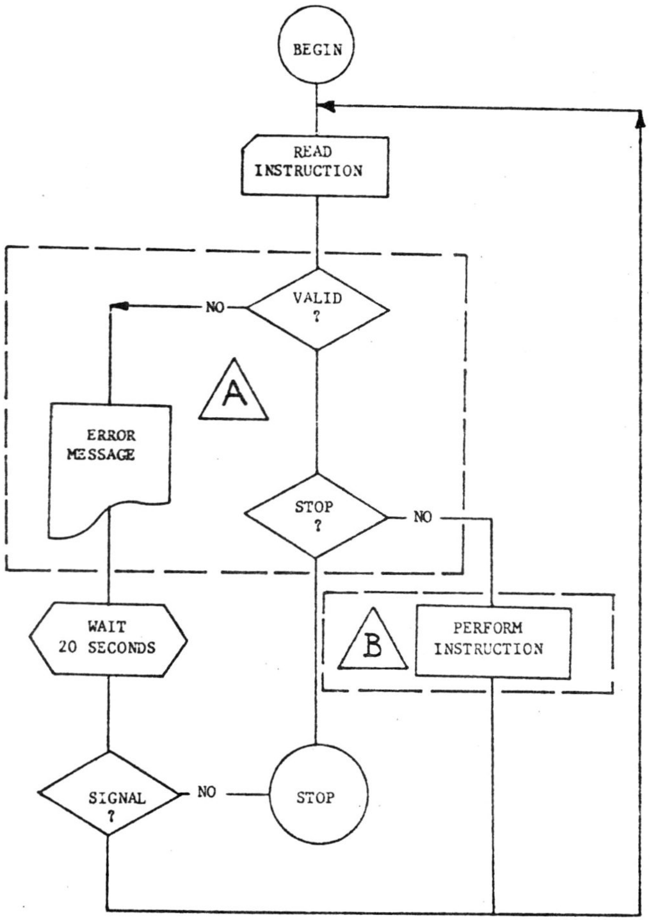

BEGIN
READ INSTRUCTION
VALID ?
NO
A
ERROR MESSAGE
STOP ?
NO
B
PERFORM INSTRUCTION
WAIT 20 SECONDS
SIGNAL ?
NO
STOP

Fig. 2a

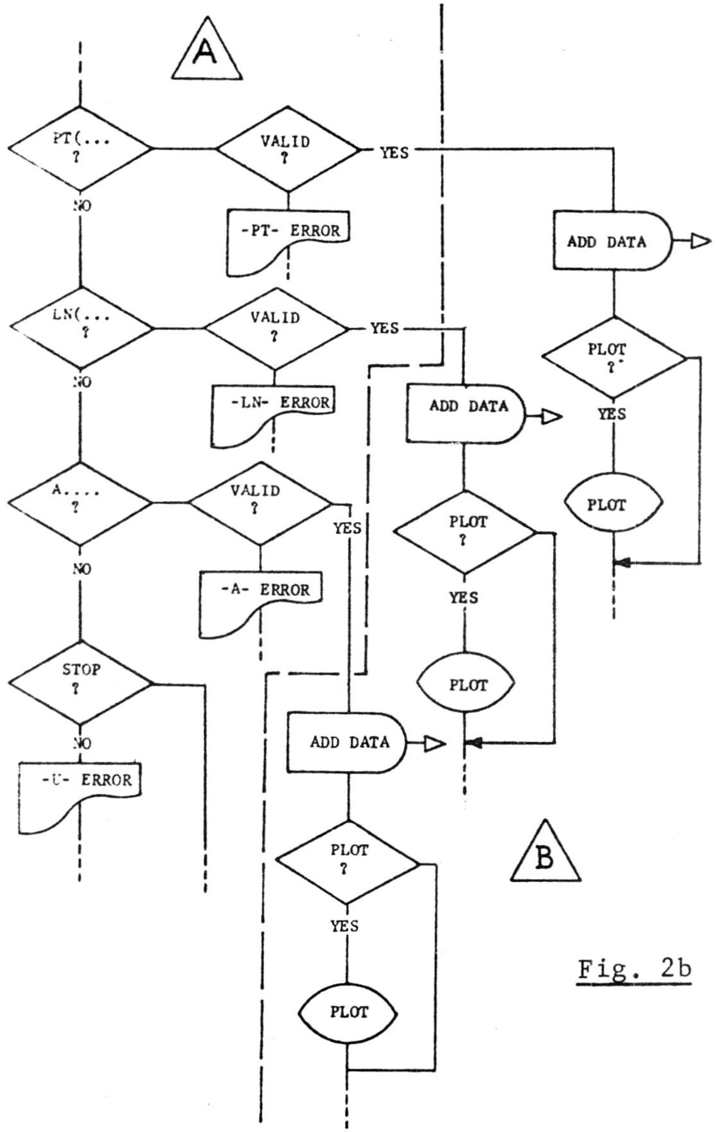

A
PT(... ?
VALID ?
YES
NO
-PT- ERROR
ADD DATA
LN(... ?
VALID ?
YES
NO
-LN- ERROR
ADD DATA
PLOT ?
YES
PLOT
A.... ?
VALID ?
YES
NO
-A- ERROR
PLOT ?
YES
PLOT
STOP ?
NO
-U- ERROR
ADD DATA
PLOT ?
YES
PLOT
B

Fig. 2b

	P	L	X	Y	Z	X_1	W_1	W'_1	X_2	W_2	W'_2	X_3	W_3	W'_3	X_4	W_4	W'_4	X_5	W_5	W'_5	X_6	W_6	W'_6	A
1	A	AB	X_A	Y_A	Z_A	$X1_A$	$W1_A$	$W'1_A$	ƀ	ƀ	ƀ	$X3_A$	$W3_A$	$W'3_A$	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	A_1
2	B	ƀ	X_B	Y_B	Z_B	$X1_B$	$W1_B$	$W'1_B$	ƀ	ƀ	ƀ	$X3_B$	$W3_B$	$W'3_B$	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ
3	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	A_3
4	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ
	.	ƀ	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	ƀ
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	ƀ
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	
19	ƀ	.	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	
20	ƀ	.	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	ƀ	
		.																						
		.																						
		.																						
39		ƀ																						
40		ƀ																						

Fig. 3

```
JØB,DGDØ
PT(A)(4.ƀƀƀƀ,-2.ƀƀƀ,2.ƀƀƀƀ)
PT(B)(10.ƀƀƀ,-5.ƀƀƀ,3.ƀƀƀƀ)
LN(AB)
AV1=AB
AV2+AB
ƀƀƀ
JØB,END
```

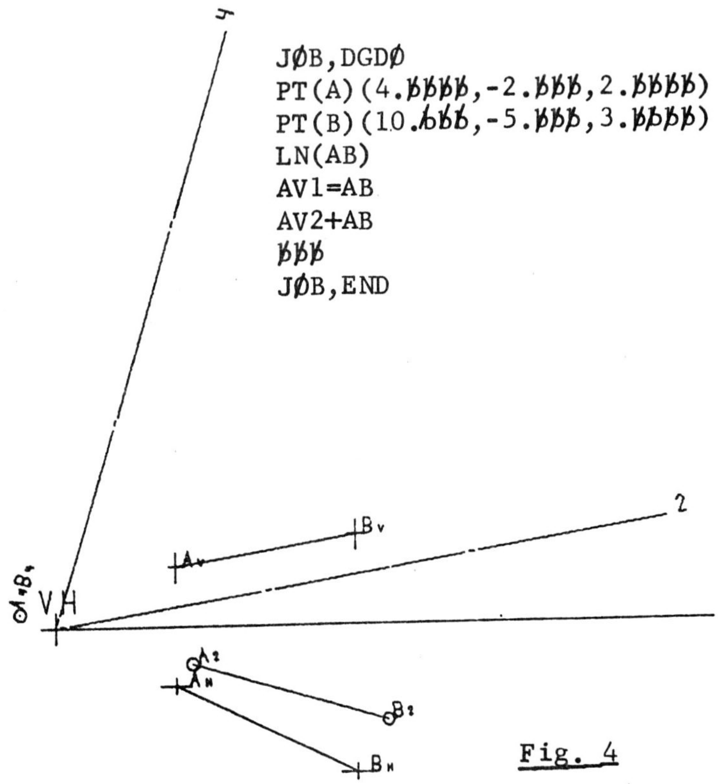
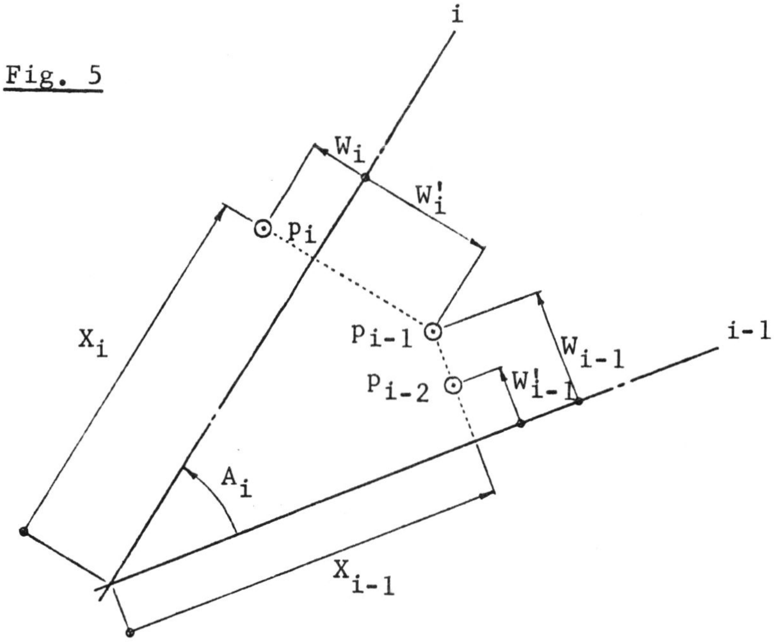
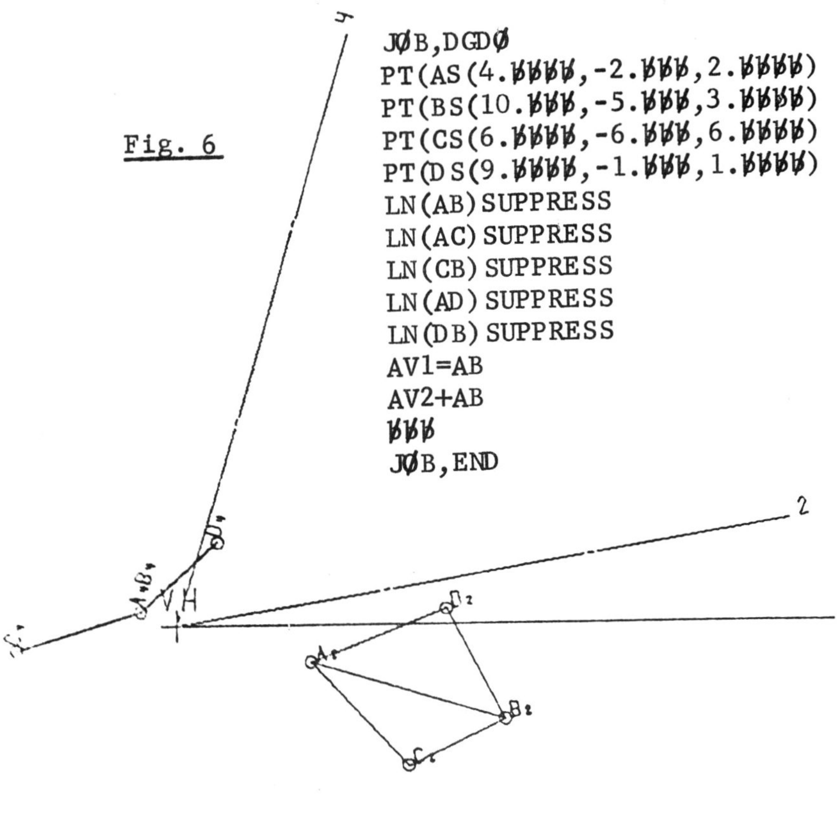


Fig. 4

<u>Fig. 5</u>



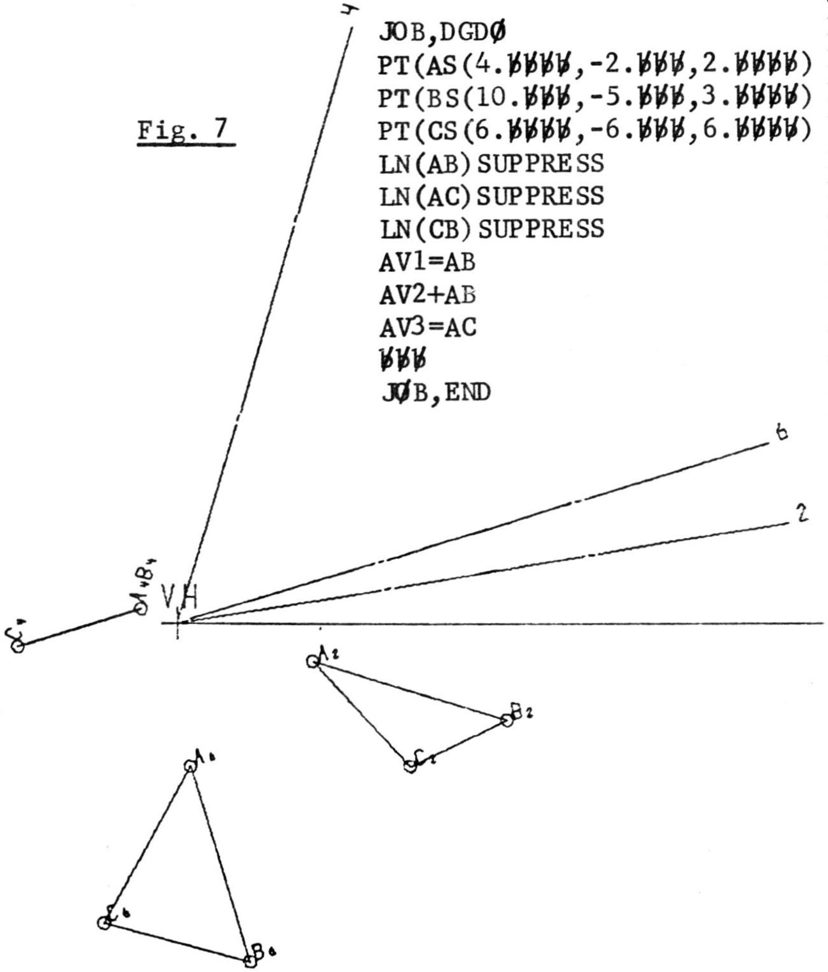
<u>Fig. 6</u>




```
JØB,DGDØ
PT(AS(4.b̸b̸b̸b̸,-2.b̸b̸b̸,2.b̸b̸b̸b̸)
PT(BS(10.b̸b̸b̸,-5.b̸b̸b̸,3.b̸b̸b̸b̸)
PT(CS(6.b̸b̸b̸b̸,-6.b̸b̸b̸,6.b̸b̸b̸b̸)
PT(DS(9.b̸b̸b̸b̸,-1.b̸b̸b̸,1.b̸b̸b̸b̸)
LN(AB)SUPPRESS
LN(AC)SUPPRESS
LN(CB)SUPPRESS
LN(AD)SUPPRESS
LN(DB)SUPPRESS
AV1=AB
AV2+AB
b̸b̸b̸
JØB,END
```

```
JOB,DGDØ
PT(AS(4.ƀƀƀƀ,-2.ƀƀƀ,2.ƀƀƀƀ)
PT(BS(10.ƀƀƀ,-5.ƀƀƀ,3.ƀƀƀƀ)
PT(CS(6.ƀƀƀƀ,-6.ƀƀƀ,6.ƀƀƀƀ)
LN(AB)SUPPRESS
LN(AC)SUPPRESS
LN(CB)SUPPRESS
AV1=AB
AV2+AB
AV3=AC
ƀƀƀ
JØB,END
```

Fig. 7




```
JØB,DGDØ
PT(P)(3.ƀƀƀƀ,-1.ƀƀƀ,1.ƀƀƀƀ)
PT(Q)(9.ƀƀƀƀ,-3.ƀƀƀ,1.ƀƀƀƀ)
PT(R)(7.ƀƀƀƀ,-6.ƀƀƀ,6.ƀƀƀƀ)
PT(E)(6.ƀƀƀƀ,-3.ƀƀƀ,3.ƀƀƀƀ)
PT(F)(8.ƀƀƀƀ,-1.ƀƀƀ,6.ƀƀƀƀ)
LN(PQ)
LN(QR)
LN(RP)
LN(EF)
AH1+PQ
AH2=QR
AH3=EF
ƀƀƀ
JØB,END
```

Fig. 8