Straight Line Walking Animation Based on Kinematic Generalization that Preserves the Original Characteristics

Hyeongseok Ko, Norman I. Badler
Department of Computer and Information Science
University of Pennsylvania, 200 South 33rd Street, Philadelphia, PA 19104-6389
Phone : 215-898-1976, 215-898-5862
e-mail : ko@graphics.cis.upenn.edu, badler@central.cis.upenn.edu

Abstract

The two most prominent problems in utilizing rotoscopy data for human walking animation can be summarized as *preservation* of the original motion characteristics in the generalization process and *constraint satisfaction*. Generalization is the process of producing, for a human figure with arbitrary (but realistic) anthropometry, the step and step length from data measured for one particular subject and step length. If we lose much of the original style in the generalization, it would be meaningless to use the measured data. We present a generalization technique that keeps the original motion characteristics. Two types of generalization are considered. One is the *anthropometry generalization*, which handles the non-uniform segment length ratios between two bodies. The other is the *step length generalization*, which changes the steps to different step lengths of the same subject. These two generalizations are combined together to generate a step of an arbitrary subject and step length. Constraint satisfaction is enforced within the generalization process.

KEYWORDS: Animation, rotoscopy, original characteristics, preservation, constraint satisfaction, human walking.

1 INTRODUCTION

A dynamic model of human walking can be described by

$$M\ddot{q} + D\dot{q} + Kq = f_{internal} + f_{others}$$

where q are the generalized coordinates, and M, D, and K are the matrices of inertia, damping, and stiffness, respectively [2, 3, 16, 6, 15, 10]. Generalized force is decomposed into two parts, the internal force (torque) $f_{internal}$ and the other forces f_{others}. The problem encountered in using the above equation in animating a *self actuated system* is that there are unknows in both sides of the equation, namely $\ddot{q}$ and $f_{internal}$. (The matrices and f_{others} are computable from the given initial (current) state q and $\dot{q}$.)

Bruderlin and Calvert built a keyframeless locomotion system [3, 2]. They generated every single frame based on both dynamics (for the movement of the underlying dynamic model of human body) and kinematics (for obtaining the detailed configuration). In the forward dynamics, they approximated $f_{internal}$ according to the general biomechanical knowledge on human walking. Their system could generate a wide gamut of walking by changing the three primary parameters and other attributes. However, the dynamic model was not easy to control, did not perform in real-time and could lead to numerical instabilities.

An enormous amount of kinematic data has been collected on human walking [12, 11], including Winter et. al's work [17, 18, 13]. A popular measuring technique for gross body motion is *rotoscoping* [18, 17], which analyzes the movement of various features or marks on the body by digitizing their successive locations over time in a film or video recording. Because the measurements are performed on a finite number of subjects (often not the same size of the figure eventually animated), without a method to generate the joint motions of other-sized figures and other step lengths, its value would be limited to direct imitation. We attempted a generalization that generates the walking step and step length of an arbitrary anthropometrically-scaled human figure from the measured step data of a particular subject and step length.

There have been other attempts [11, 12, 7, 9] to obtain generic properties of human walking. These properties, which are basically the average of the subjects considered, do not provide exact data for one particular subject and step. Therefore, an *ad hoc* correction may be required to animate a specific figure in stepping a specific step length.

If it is not accurate, the generalization (of either rotoscoped data, or accumulated knowledge) may lead to a constraint violation or discontinuity between the steps. Just enforcing the constraints, however, may lead to another problem: losing the original motion characteristics. Our research has sought a kinematic generalization that doesn't violate any obvious constraints and at the same time preserves the original motion characteristics.

Boulic et al. tried a generalization of experimental data based on the normalized velocity of walking [1]. Their generalization could produce the parameters which might violate, in its direct application, some of the constraints imposed on walking. They overcame this problem through a correction phase based on inverse kinematics. To preserve the original characteristics of the walking data, they introduced the *coach concept*, which basically chooses the one among the multiple inverse kinematics solutions that is closest to the original motion.

In our approach, the constraints are strongly enforced within the generalization process, obviating the correction phase: there is no *skidding* nor *penetration* of the supporting foot on or through the supporting plane. Also, the original locomotion style is *explicitly* maintained.

2 BASIC IDEA OF CHARACTERISTIC PRESERVATION

Suppose a measured data set $W(S_1, sl_1)$ of the subject S_1 and the step length sl_1 is given. Our goal in this paper is to generate another data set $W(S_2, sl_2)$ of arbitrary subject S_2 and step length sl_2. In this way, from the data of one particular subject and step, we can produce steps of any subject and step length.

When another step data $W(S_3, sl_3)$ is to be generated, other approaches may use the original measured data $W(S_1, sl_1)$ rather than the generalized data $W(S_2, sl_2)$ as the input of the generalization process, because some characteristics of the original motion may have been lost in producing the first generalization. If both $W(S_1, sl_1)$ and $W(S_2, sl_2)$ produce the same result, however, then we may consider the original characteristics of $W(S_1, sl_1)$ to be maintained in $W(S_2, sl_2)$ during the generalization.

Furthermore, if the above is true for any S_2, S_3 and sl_2, sl_3 (*transitive*), then $\breve{W}(S_n, sl_n)$ after the series of generalizations ($W(S_1, sl_1)$, $W(S_2, sl_2)$, ..., $\breve{W}(S_n, sl_n)$) will be the same as $\tilde{W}(S_n, sl_n)$ after the direct generalization ($W(S_1, sl_1)$, $\tilde{W}(S_n, sl_n)$). Under the transitivity, we can keep any one of the intermediate results instead of the original measured data for further generalization. The transitivity will be used as the measure of characteristic preservation. We will show in the subsequent sections that our generalization scheme is indeed transitive.

One merit of our generalization method is that it can be extended *incrementally.* Because it basically *imitates* the original motion, we can *simulate different locomotion styles* by acquiring multiple sets of measurements. Thus, in one scene, several people can walk in their own individual locomotion patterns. Moreover, the underlying pattern can be smoothly interpolated from one data set to another if the animator wants to vary the walk style itself.

3 DEFINITIONS

A *timed sequence* Q is a set of 2-tuples

$$Q = \{(t_i, v_i) \mid i = 1, \ldots, n\} \qquad (1)$$

where each t_i is a real number with $t_i < t_{i+1}$ for $i = 1, \ldots, n-1$, and v_i can be any dimensional vector but should be the same dimensional for every i. For any timed sequence Q, we can define the function *interpolation* as:

$$interpolation(Q, t) = v_i + \frac{t - t_i}{t_{i+1} - t_i}(v_{i+1} - v_i), \quad t_i \le t \le t_{i+1}. \qquad (2)$$

At a certain moment, if a leg is between its own heelstrike (beginning) and the other leg's heelstrike (ending), it is called the *stance leg.* If a leg is between the other leg's heelstrike(beginning) and its own heelstrike (ending), it is called the *swing leg.* For example, in Figure 1, left leg is the stance leg during the interval 1, and right leg is the stance leg during the interval 2. Thus at each moment we can refer to a specific leg by either stance or swing leg with no ambiguity. The joints and segments in a leg will be referred to using prefixes *swing* or *stance.* For example, swing ankle is the ankle in the swing leg. (In the literature, the stance and the swing phases are longer and shorter than one step duration, respectively. Our definition of the prefixes stance and swing is solely for clear designation of the legs at any moment.)

Let HSM^- be the Heel Strike Moment just before the current step, HSM^+ be the Heel Strike Moment right after the current step, which is one step after HSM^-, FGM be the moment when the stance foot is put flat on the ground (Flat Ground Moment), MOM be the Meta Off Moment when the toes begin to be off the floor and rotate around the tip of the toe, and TOM be the Toe Off Moment.

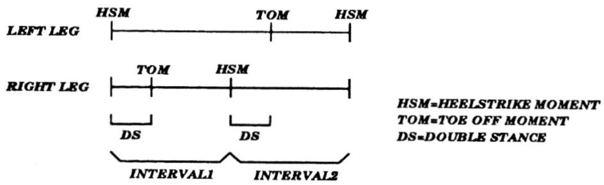

Figure 1: The Phase Diagram of Human Walk

The *anthropometry* $B(S)$ of a subject S is simply the m-tuple $(l_1, \ldots, l_m)$, where l_i is the length of the ith segment and m is the total number of the segments. We say $S_2 = \alpha S_1$ *iff* $B(S_2) = \alpha B(S_1)$. In this case S_2 is also denoted as αS_1.

Among the anthropometry components, the one that mostly affects the lower body movement in walking would be the length of the leg. The *leg length* $ll(S)$ of the subject S is defined to be the sum of the lengths of the thigh and the calf. We say $S_2 \approx \alpha S_1$ *iff* $ll(S_2) = ll(\alpha S_1)$. In this case, S_2 is denoted as $\tilde{\alpha} S_1$, and we say

$$\alpha = \frac{S_2}{S_1} \tag{3}$$

Note that $S_2 \approx \alpha S_1$ and $\alpha = \frac{S_2}{S_1}$ hold also when $S_2 = \alpha S_1$. The *walk condition* is simply the tuple (S, sl) of the subject and the step length.

The *walk data* $W(S, sl)$ of the subject S in stepping the step length sl is defined as the collection of the six timed sequences

$$W(S, sl) = \{H, F_1, F_2, F_3, M, A\} \tag{4}$$

where each element represents hip trajectory, foot sole angle (simply *foot angle* later on) trajectories during $[HSM^-, FGM]$, $[FGM, TOM]$, and $[TOM, HSM^+]$, the *meta angle* trajectory, and the ankle trajectory, respectively. The meta angle is the angle between the floor and the toes during the meta off phase [3, 2].

Note that H and A are in the Cartesian space, and the other elements are in the joint space. H and $F_i, (i = 1, 2, 3)$ govern the stance leg of the current step, from the start to the end of the step. M is for the current stance leg from the MOM until the TOM of the next step. A is for the swing leg.

All the time values used in defining the timed sequences are normalized according to the step duration. For example, $HSM^- = 0$ and $HSM^+ = 1$. Together with the function *interpolation*, $W(S, sl)$ provides enough information to generate the lower body movement during one step.

In the following sections, we will assume that we have the measured data $\bar{W}(S^*, sl^*)$ of the subject S^* in stepping the step length sl^*, which will be called *prototype walk data*.

4 PROPERTIES OF OUR STEP LENGTH GENERALIZATION

We will denote the whole step length generalization process by ϕ_ρ. It can be interpreted as an operator (subroutine) whose input is walk data $W(S, sl)$ and its output is another walk of the same subject with step length ρ times that of the original one. It can be compactly written as

$$W(S, \rho sl) = \phi_\rho(W(S, sl)). \tag{5}$$

The details of it is defined in Appendix B.

Theorem 1 *Our step length generalization can be composed in the following way for any positive number* ρ_1 *and* ρ_2.

$$\phi_{\rho_1\rho_2} = \phi_{\rho_2} \circ \phi_{\rho_1} \tag{6}$$

□

(The proofs of the theorems in this paper can be found in [8].)

Corollary 1 *For any two step length generalizations* ϕ_{ρ_1} *and* ϕ_{ρ_2}, *their composition is commutative.*

$$\phi_{\rho_2} \circ \phi_{\rho_1} = \phi_{\rho_1} \circ \phi_{\rho_2} \tag{7}$$

□

Corollary 2 *For any three step length generalizations* ϕ_{ρ_1}, ϕ_{ρ_2}, *and* ϕ_{ρ_3}, *their composition is associative.*

$$\phi_{\rho_1} \circ (\phi_{\rho_2} \circ \phi_{\rho_3}) = (\phi_{\rho_1} \circ \phi_{\rho_2}) \circ \phi_{\rho_3} \tag{8}$$

□

Based on the discussion in Section 2 we make the following definition:

Definition 1 *A walk data generalization is said to be* characteristic preserving *if the it is transitive.* □

A question at this point is whether ϕ is characteristic preserving. That is, for any step lengths sl_1, sl_2, and sl_3, is $\phi_{\frac{sl_3}{sl_2}} \circ \phi_{\frac{sl_2}{sl_1}}$ equal to $\phi_{\frac{sl_3}{sl_1}}$? The following theorem answers the question.

Theorem 2 *Our walk data generalization* ϕ *on the step length is characteristic preserving.* □

5 GENERALIZATION AMONG DIFFERENT BODIES

Let us imagine two human figures A and B. B's kinematic property is α times that of A's, in every aspect segmentwise. That is, $B = \alpha A$. In this situation, if B is walking at a step length that is α times as long as that of A's, what would be the joints angles of B compared to the corresponding joint angles of A? We assume that they are the *same*. Therefore if we have data for A at the step length sl, we can use it for B at the step length αsl. This will be called the *similarity assumption* later on.

The above is justifiable by Murray et al.'s experiment [12]. They divided 60 subjects into 3 groups (20 subjects in each group) according to their height: tall, medium, and short. Each subject was trained to walk freely as they usually do. A significant correlation between the height and the stride length was found. Table 1 shows the stride length is linearly related with the height. In this experiment they observed that *there were no significant differences* in the major joint (hip, knee, ankle) angles among the groups, which supports the above similarity assumption.

The flexion angle of the hip of the group Short was slightly bigger than the other two groups. That phenomenon can be explained by the slightly increasing ratio values in the above table. Because people live in a community, there tends to be a regression effect in walking. The shorter group's stride length relative to their height tends to be longer than that of the longer group. If the ratio value in the experiment was maintained constant, then the hip flexion of the Short might be closer to the other groups.

There have been many trials to find the relative size of the segments in human body [4, 5]. Since such information depends on the individuals, the results depended on the sampled subjects from which the statistics were computed. The sampling may differ among the research groups. Therefore in using rotoscopy data for human walking animation in particular, the model used in the animation is more likely to be different from the subject on which the measurement was performed, not only in the total size but also in the individual ratios of the corresponding segments. So the similarity assumption alone can not cover the variety of locomotion phenomenon for general anthropometry.

The walk data $W(\alpha S, \alpha sl)$ is simply derivable from $W(S, sl)$, based on the similarity assumption. Generally speaking, the Cartesian quantity is scaled by α and the angular quantity remains the same. However, if the scale is not the same between the corresponding segments, the similarity assumption can not be applied directly. Let the walk data $W(S, sl)$ of the subject S be given at the step length sl. The derivation of the walk data of an arbitrary subject S' and step length αsl, where $S' \approx \alpha S$, is included in Appendix C. The whole process will be denoted by $\psi_{S_1}^{S_2}$, which maps the walk data $W(S_1, sl)$ to $W(S_2, \frac{ll(S_2)}{ll(S_1)} sl)$. I.e,

$$W(S_2, \frac{ll(S_2)}{ll(S_1)} sl) = \psi_{S_1}^{S_2}(W(S_1, sl)) \qquad (9)$$

Theorem 3 *For any subjects $S, S_1, and S_2$,*

$$\psi_{S_1}^{S_2} \circ \psi_{S}^{S_1} = \psi_{S}^{S_2} \qquad (10)$$

Therefore the anthropometry generalization is characteristic preserving. □

6 COMBING THE TWO TYPES OF GENERALIZATION

To have a full generalization, the two kinds of generalization, namely, the step length generalization and the anthropometry generalization, should be combined together. Let's suppose the original walk is (S_1, sl_1), and the desired walk is (S_2, sl_2). We can apply the anthropometry generalization first

$$W(S_2, \frac{S_2}{S_1} sl_1) = \psi_{S_1}^{S_2}(W(S_1, sl_1)) \qquad (11)$$

and then apply the step length generalization to get the final result

$$W(S_2, sl_2) = \phi_{\frac{S_1}{S_2}\frac{sl_2}{sl_1}}(W(S_2, \frac{S_2}{S_1} sl_1)). \qquad (12)$$

Another way around is to apply step length generalization first

$$W(S_1, \frac{S_1}{S_2} sl_2) = \phi_{\frac{S_1}{S_2}\frac{sl_2}{sl_1}}(W(S_1, sl_1)) \qquad (13)$$

and then apply the anthropometry generalization,

$$W(S_2, sl_2) = \psi_{S_1}^{S_2}(W(S_1, \frac{S_1}{S_2} sl_2)). \qquad (14)$$

One obvious question here is which way is correct or better. It seems desirable that the order of the applications of the generalization does not affect the result. In fact, our generalization algorithm does have that property.

Theorem 4 *In applying the generalizations ϕ and ψ defined in the previous sections, the order of the application does not affect the final result. I.e., for any*

Group	Mean Height(in)	Stride Length(in)	Ratio(Stride/Height)
Tall	72.2	63.98	0.886
Medium	69.1	61.50	0.890
Short	66.0	59.37	0.899

Table 1: Murray's Experiment

walk data $W_1 = W(S_1, sl_1)$, and for any walk condition (S_2, sl_2),

$$\phi_{\frac{S_1}{S_2}\frac{sl_2}{sl_1}}(\psi_{S_1}^{S_2}(W_1)) = \psi_{S_1}^{S_2}(\phi_{\frac{S_1}{S_2}\frac{sl_2}{sl_1}}(W_1)) \quad (15)$$

or simply

$$\phi_{\frac{S_1}{S_2}\frac{sl_2}{sl_1}} \circ \psi_{S_1}^{S_2} = \psi_{S_1}^{S_2} \circ \phi_{\frac{S_1}{S_2}\frac{sl_2}{sl_1}} \quad (16)$$

□

We have shown (theorems 2 and 3) both types of generalization, i.e., the step length generalization ϕ and the anthropometry generalization ψ, are characteristic preserving when they are applied *homogeneously*. One obvious question here is whether it is characteristic preserving even when they are mixed up together. In fact, our generalization scheme does have that property.

Theorem 5 *Let $WC_1 = (S_1, sl_1)$, $WC_2 = (S_2, sl_2)$, and $WC_3 = (S_3, sl_3)$ be three arbitrary walk conditions. Let*

$$\tau_{12} = \psi_{S_1}^{S_2}\phi_{\frac{S_1}{S_2}\frac{sl_2}{sl_1}} \quad (17)$$

$$\tau_{23} = \psi_{S_2}^{S_3}\phi_{\frac{S_2}{S_3}\frac{sl_3}{sl_2}} \quad (18)$$

$$\tau_{13} = \psi_{S_1}^{S_3}\phi_{\frac{S_1}{S_3}\frac{sl_3}{sl_1}} \quad (19)$$

be the combined generalizations that try to transform WC_1 to WC_2, WC_2 to WC_3, and WC_1 to WC_3, respectively. Then

$$\tau_{23} \circ \tau_{12} = \tau_{13} \quad (20)$$

Therefore the combined generalization is characteristic preserving.

7 CONCLUSION

The generalization algorithm is implemented in *Jack*™ [14]. In the implementation, the arm swing is done by a kinematic function that depends on the leg movement. The accompanying animation is based on the measured data from [17]. The following table shows the comparison between the subject measured and the figure animated, in both their step lengths and anthropometry. Our animation was quite successful for all the anthropometric and step length differences between the subject and the figure.

Even though the definition of the preservation was aimed for the animation, we should note the difference of its meaning, in the *mathematics space* and in the *animation space*. If the desired step is too different from the originally measured one, even though the characteristic is well preserved in mathematical way, it has less meaning in generating that step based on the original one. For example, if the subject $2S$ tries to imitate the walk of (S, sl) in stepping $0.1sl$, the goals of imitating and achieving the given step length will be in total conflict. The characteristics are well preserved in the mathematical space: further generalization from $(2S, 0.1sl)$ to $(2S, 2sl)$ will produce a similar walking pattern as (S, sl). However, to get a better animation of $(2S, 0.1sl)$, we need a measurement of S at a smaller step.

We have demonstrated a new approach in generalizing rotoscopy data for human walking animation, which promises good results by the nature of the method. From the measured data of *one step* of a particular subject, we can generate reasonable steps of any step length and for any anthropometry. We can extend our system to simulate multiple walking styles by acquiring other measurements. Characteristic preservation is a new criteria for determining the *quality of generalization*.

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	Subject (Measured)	Figure (Animated)	Ratio (Figure/Subject)
step length	70.69	variable	variable
ankle to heel	7.629	13.49	1.77
ankle to ball	12.20	17.55	1.44
heel to ball	15.89	15.19	0.96
ball to toe	7.095	7.200	1.01
ball angle	0.4842	0.8378	1.73
heel angle	0.8400	1.312	1.56
ankle angle	1.818	0.9916	0.55
heel to N	5.093	3.450	0.67
ankle to N	5.680	13.04	2.30
ball to N	10.80	11.74	1.09
shin	38.05	34.53	0.91
thigh	35.85	40.72	1.14
leg length	73.90	75.25	1.02

Table 2: The Comparison between the Measured Subject and the Animated Figure (Unit:inches,radians)

9 APPENDICES

A μ FUCTIONS

The knee angle at the heel strike moment is defined as

$$\mu_1(S, sl) = -\alpha_1(\frac{ll(S)}{ll(S^*)}sl - sl^*) + \mu_1^* \quad (21)$$

where μ_1^* is the knee angle at the heel strike moment in the prototype data. We can increase or decrease α_1 within the range [0,0.3] without affecting the preservation property of our generalization. This specific interval is based on Inman's work [7].

The foot angle at the heel strike moment is defined as

$$\mu_2(S, sl) = \alpha_2(\frac{ll(S)}{ll(S^*)}sl - sl^*) + \mu_2^* \quad (22)$$

where μ_2^* is the foot angle at the heel strike moment in the prototype walk data, and α_2 is a positive constant we can control.

The foot angle at the toe off moment is defined as

$$\mu_3(S, sl) = \alpha_3(\frac{ll(S)}{ll(S^*)}sl - sl^*) + \mu_3^* \quad (23)$$

where, μ_3^* is the foot angle at the toe off moment in the prototype walk data, and α_3 is a positive constant we can control.

The meta angle at the toe off moment is defined as

$$\mu_4(S, sl) = \alpha_4(\frac{ll(S)}{ll(S^*)}sl - sl^*) + \mu_4^* \quad (24)$$

where, μ_4^* is the meta angle at the toe off moment in the prototype walk data, and α_4 is a positive constant we can control.

B STEP LENGTH GENERALIZATION: ϕ_ρ

The six components of H, F_1, F_2, F_3, M, and A of $W(S, \rho sl)$ are derived from $W(S, sl)$. The timed sequences with superscript 1 are for the walk data $W(S, sl)$, and those with superscript 2 are for $W(S, \rho sl)$.

$$H^1 = \{(t_i, x_i, y_i) \mid i = 1, \ldots, n\}, \quad (25)$$

$$H^2 = \{(t_i, \rho' x_i, y_i') \mid i = 1, \ldots, n\} \quad (26)$$

where

$$y_i' = y_i + (1 - t_i)(\tilde{y}_1 - y_1) + t_i(\tilde{y}_n - y_n) \quad (27)$$

and

$$\rho' = \frac{sl_{before} + \rho sl}{sl_{before} + sl}. \quad (28)$$

sl_{before} is the step length of the previous step.

$$F_1^1 = \{(t_{1i}, f_{1i}) \mid i = 1, \ldots, n_1\}, \quad (29)$$

$$F_2^1 = \{(t_{2i}, f_{2i}) \mid i = 1, \ldots, n_2\}, \quad (30)$$

$$F_3^1 = \{(t_{3i}, f_{3i}) \mid i = 1, \ldots, n_3\}, \quad (31)$$

with $t_{1i_{n_1}} = t_{21}$ and $t_{2i_{n_2}} = t_{31}$.

$$F_1^2 = \{(t_{1i}, f_{1i}\frac{\mu_2(S, sl_{before})}{f_{11}}) \mid i = 1, \ldots, n_1\}, \quad (32)$$

$$F_2^2 = \{(t_{2i}, f_{2i}\frac{\mu_3(S, \rho sl)}{f_{2n_2}}) \mid i = 1, \ldots, n_2\}, \quad (33)$$

$$F_3^2 = \{(t_{3i}, f_{3i}') \mid f_{3i}' = f_{3i} + (1 - t_{3i}')d_1 + t_{3i}'d_2,$$

$$i = 1, \ldots, n_3\} \quad (34)$$

where $t'_{3i} = \frac{t_{3i}}{HSM^+ - TOM}$, $d_1 = \mu_3(S, \rho sl) - f_{31}$, and $d_2 = \mu_2(S, \rho sl) - f_{3n_3}$.

$$M^1 = \{(t_i, m_i) \mid i = 1, \ldots, n_m\}, \quad (35)$$

$$M^2 = \{(t_i, m_i \frac{\mu_4(S, \rho sl)}{m_{n_m}}) \mid i = 1, \ldots, n_m\}. \quad (36)$$

Let the coordinates of the ankle just before the TOM and right after the HSM^+ of the step length sl be (b_{1x}, b_{1y}) and (b_{2x}, b_{2y}), respectively; likewise, those of the step length ρsl be (b_{3x}, b_{3y}), and (b_{4x}, b_{4y}), respectively. These are computable from the μ-functions defined in Appendix A.

$$A^1 = \{(t_i, x_i, y_i) \mid i = 1, \ldots, n_a\}, \quad (37)$$

$$A^2 = \{(t_i, x'_i, y'_i) \mid i = 1, \ldots, n_a\}, \quad (38)$$

where

$$x'_i = b_{3x} + \frac{b_{4x} - b_{3x}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (39)$$

and

$$y'_i = y_i + (1 - t'_i)d_1 + t'_i d_2 \quad (40)$$

with $d_1 = b_{3y} - y_1$, $d_2 = b_{4y} - y_{n_a}$, and $t'_i = \frac{t_i - TOM}{HSM^+ - TOM}$.

C ANTHROPOMETRY GENERALIZATION: ψ_α

The timed sequences with superscript 1 are for the walk data $W(S, sl)$, and those with superscript 2 are for $W(S', \alpha sl)$, where $\alpha = \frac{S'}{S}$. The angular trajectories, F_1, F_2, F_3, M are invariant in the anthropometry generalization.

$$H^1 = \{(t_i, x_i, y_i) \mid i = 1, \ldots, n\}, \quad (41)$$

$$H^2 = \{(t_i, x'_i, y'_i) \mid i = 1, \ldots, n\}, \quad (42)$$

where

$$x'_i = \alpha x_i \quad (43)$$

$$y'_i = \alpha y_i + \Delta a N. \quad (44)$$

$\Delta a N$ is the difference between the ankle heights of S' and αS (positive if S' is higher).

Let the coordinates of the ankle just before the TOM and right after the HSM^+ of the subject S with the step length sl be (b_{1x}, b_{1y}) and (b_{2x}, b_{2y}), respectively; likewise, those of the subject S' with step length αsl be (b_{3x}, b_{3y}), and (b_{4x}, b_{4y}), respectively. As in the step length generalization, b_{4x} is determined so that the resulting step length is αsl.

$$A^1 = \{(t_i, x_i, y_i) \mid i = 1, \ldots, n_a\}, \quad (45)$$

$$A^2 = \{(t_i, x'_i, y'_i) \mid i = 1, \ldots, n_a\}, \quad (46)$$

where

$$x'_i = b_{3x} + \frac{b_{4x} - b_{3x}}{b_{2x} - b_{1x}}(x_i - b_{1x}) \quad (47)$$

and

$$y'_i = \alpha y_i + (1 - t'_i)d_1 + t'_i d_2 \quad (48)$$

with $d_1 = b_{3y} - \alpha y_1$, $d_2 = b_{4y} - \alpha y_{n_a}$, and $t'_i = \frac{t_i - TOM}{HSM^+ - TOM}$.

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