# digital filtering for boundary smoothing IN DIGITAL THEMATIC MAPS 

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## ABSTRACT

This paper is concerned with the problem of smoothing digital thematic maps. The method proposed uses a digital filtering approach, whereby a weighted neighborhood operator is scanned over the classified image calculating the most likely value for each pixel of the image. The filtered image is then processed by a previously developed minimal area algorithm. Different weighting schemes and neighborhood sizes are proposed and applied to a sample image and the results presented.

FILTRAGE NUMERIQUE POUR LE LISSAGE DES CONTOURS DANS LES CARTES THEMATIQUES

## RESUME

La présente communication a trait au problème du lissage numérique des courbes de niveau des cartes thématiques. La méthode proposée fait appel à un filtrage numérique dans lequel un opérateur à voisinage pondéré explore toute la surface de l'image préparée, en calculant la valeur la plus probable de chaque pixel de l'image. Cette image filtrée est ensuite traitée par un algorithme de superficie minimale, existant déjà. On propose différentes structures de pondération et diverses tailles de voisinages que l'on applique à un exemple d'image; on présente les résultats obtenus.

## Introduction

Digital thematic maps produced from satellite imagery are important tools in the analysis of land regions for applications such as forest management and environmental research. Each pixel (picture element) in a satellite picture is classified according to its representative wavelength (spectral signature) in the visible spectrum independently from surrounding pixels. Each class is then assigned a number and the classified picture stored.

Forest inventory maps so produced are rarely useful in their original form. Instead, a user is interested in regions at a macro level. This requires that a picture be processed to produce relatively large homogeneous regions with smooth boundaries, similar to those that would be produced by a human interpreter. An algorithm [1] has been developed which groups pixels into regions and renames the smaller regions so that each region is larger than a preset minimum. That method successfully results in relatively large homogeneous regions. Unfortunately, the regions do not have smooth boundaries, and do not approach what would be produced by manual methods.

This paper describes an algorithm which can be applied to the other problem: smoothing region boundaries such that few discontinuities exist.

Terminology
The following terms will be used throughout this paper:

A picture $P$ is a 2-dimensional matrix of class numbers.

A pixel $p(i, j)$ is a cell at location (i,j) in $P$ which contains a class number.

Two pixels $p$ and $q$ are connected iff they are horizontally, vertically, or diagonally adjacent in the picture matrix.

Pixels $p$ and $q$ are chain-connected iff there exists a sequence of pixels $p=p(1), p(2), \ldots, p(k)=q$ such that $p(i)$ and $p(i+1)$ are connected for all $i=1,2, \ldots, k-1$.

A region of $P$ is a set of pixels such that any 2 pixels in the set are chain-connected.

The first order neighborhood (NHD=1) of $p(i, j)$ is the set of pixels $p(k, l)$ for all values of $k=i-1, i, i+1$ and l=j-1, i, j+l.

The $n-t h$ order neighborhood (NHD=n) of $p(i, j)$ is the set of pix̀ls $\overline{p(k, l)}$ for all values of $k=i-n, i=n+1, \ldots$, $i+n$ and $l=j-n, j-n+1, \ldots, j+n$.

An $n$-th order weighted neighborhood matrix (WHND) is a square mátrix of size $2 * n+1$ whose elements are arbitrary numbers. The elements of WNHD are chosen to represent a weighting factor according to their proximity to the centre element of WNHD. WHND is symmetric on both diagonals.

A class weight vector (wclass) is a vector of arbitrary numbers chosen to represent weights of possible pixel classes. wclass(i) is the weight of class i.

For a given pixel $p(i, j)$, the class measure of $p(i, j)$ is defined as a function of wclass $(\overline{p(i, j)})$ and of wHND such that $p(i, j)$ corresponds to the centre element of WNHD. WNHD can be thought of as a template which overlays $p$ such that its centre element overlays $p(i, j)$. For a given pixel $p(i, j)$ in $P$, there is a class measure for each class in the neighborhood of $p(i, j)$.

The majority class of a pixel $p(i, j)$ is that class for which the class measure is the greatest.

## Methodology

The algorithm presented for boundary smoothing is based on the theory that the class of each pixel in a picture can be converted to a new class in such a way as to "smooth" or "generalize" the regions removing discontinuities in their boundaries. Basically, the algorithm is as follows:

1. Each pixel $p(i, j)$ in a picture $P$ is examined together with its neighborhood, the wclass found, and the majority class determined.
2. The class of pixel $p(i, j)$ becomes the majority class. This new class may very well be the same as the original class, in which case the class of $p(i, j)$ does not change.

The previously presented [1] minimum area algorithm eliminates isolated pixels by considering a first order neighborhood and converting to the majority class. The current algorithm generalizes that process by converting all pixels to a majority class determined by a higher order neighborhood. The majority class is that class for which the class measure is the greatest. The class measure is determined for each class occurring in the neighborhood of a pixel as a function of class weight and positional weight in the neighborhood. Class weights throughout this project
were all set to 1 . Positional weight is found in WHND, which can be considered as a square template whose centre element overlays the current pixel p(i,j). The algorithm then scans the pixels in the neighborhood of $p(i, j)$ which correspond to the cells of WNHD.

Specifically, the algorithm is as follows: p(i,j) is the current pixel in $p$. Let $n$ be the neighborhood order ( $N H D=n$ ) so that WHND is of size $2 * n+1$. The algorithm looks at the set of pixels $p(k, l)$ in $P$ such that:

$$
\begin{array}{lll}
k=i-n, & i-n+1, & \ldots, \\
l+n \\
l=j-n, & j-n+1, & \cdots, \\
j+n
\end{array}
$$

The class weight of each $p(k, l)$ is multiplied by WHND(r,s) where:

$$
\begin{aligned}
& r=i-n-1+k \text { and } \\
& s=j-n-1+1 .
\end{aligned}
$$

This result is then added to the class measure for the class of $p(k, l)$. When the whole neighborhood has been scanned, the class for which the class measure is greatest becomes the new class for $p(i, j)$. The algorithm then considers the next pixel in $p, i . e ., p(i+1, j) . \quad P$ is scanned left to right, top to bottom.

The size and contents of WHND have a marked effect on the smoothed picture $P^{\prime}$. It is logical to assume that the further a neighborhood pixel is away from the current pixel in $P$, the less weight its location will have in determining the majority class. Thus, neighborhood weight assignment functions were developed in which an arbitrary weight is assigned the center element of WHND and weights then assigned to the other elements based on their distance from the center element and the weight of the center element. If WHND (i,j) is the center element with weight $x$, then the weight of any other element WHND(k,l) is a function of $x$, $i-k$, and j-l. That is,

$$
\begin{aligned}
& \text { WHND }(k, l)=f(x, z l, z 2) \\
& \text { where: } \\
& z 1=i-k \text { and } \\
& z 2=j-1 .
\end{aligned}
$$

The following weight assignment functions were used in defining WHND:

```
a. WHND (k,l)=x-|zl|-|z2|
b. WHND (k,l) = x - SQRT(zl**2 + z2**2)
c. WHND (k,l) = x / (SQRT (zl**2 + z2**2) + l)
d. WHND (k,l) = x / (2*SQRT(zl**2 + z2**2) + l)
```

where:

```
x = an arbitrary weight assigned to the center
element of WHND
zl = i-k
z2 = j-1
i = the row number of the center element of WHND
j = the column number of the center element of
WHND
n = neighborhood order
k = 1, 2, ..., 2*n+1
l = 1, 2, .... 2*n+1.
```

Throughout this project $x$ was set to 2 * $n+1$.
Results
Many pictures and tables were generated during the project with results being consistent overall. Neighborhoods of order 1 to 5 were tried for each function a to d above. It was felt that NHD=5 (which means WHND is an 11 x 11 matrix) already consumes excessive CPU time, and, since processing time rises exponentially with NHD, larger orders were not feasible on today's computers. These trials determined the effect of the smoothing algorithm on the original picture.

In addition, it was decided to experiment on pictures using combinations of the smoothing algorithm and the minimum area algorithm.

The 4 pictures found in Figures 1 thru 4 were chosen to illustrate the findings. Figure 1 is the original classified image Pl. Figure 2 (P3) was obtained starting with Pl and using NHD $=3$ and

WHND $=\mathrm{X}-\mathrm{SQRT}(\mathrm{zl} * * 2+z 2 * * 2)$.
Figure 3 containing P6 was obtained starting with Pl and using

$$
\text { NHD }=5 \text { and WHND }=X /(2 * \operatorname{SQRT}(z 1 * * 2+z 2 * * 2)+1) \text {. }
$$

Figure 4 contains P9 which was obtained by starting with Pl and using the minimal area algorithm with a minimal area of 50, and then applying the smoothing with NHD $=3$, and

WHND $=X /(2 * \operatorname{SQRT}(z 1 * * 2+z 2 * * 2)+1)$.
The conversion matrices for P3, P6 and P9 are shown in Tables I, II and III respectively, The region information in P3 and P9 is summarized in Tables IV and $V$ respectively.

Table I. Class Conversions for P3.

|  |  | TO |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 |  |
|  | 1 | 711 | 47 | 341 | 692 | 1146 |  |
| 2 | 19 | 2618 | 66 | 192 | 273 |  |  |
| FROM | 3 | 5 | 50 | 18185 | 4135 | 260 |  |
|  | 4 | 62 | 41 | 2805 | 26614 | 318 |  |
|  | 5 | 293 | 381 | 514 | 489 | 5279 |  |

Table II. Class Conversions for P6.

|  |  | TO |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | 1 | 2 | 3 | 4 | 5 |
|  | 1 | 653 | 39 | 376 | 736 | 1133 |
| 2 | 17 | 2514 | 102 | 224 | 311 |  |
| FROM | 3 | 1 | 55 | 18083 | 4217 | 279 |
|  | 4 | 38 | 28 | 2639 | 26813 | 322 |
|  | 5 | 150 | 347 | 543 | 525 | 5391 |

Table III. Class Conversions for P9.

|  | TO |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1 | 1 | 2 | 3 | 4 | 5 |
|  | 832 | 0 | 16 | 90 | 223 |  |
| FROM | 3 | 0 | 2909 | 5 | 35 | 35 |
|  | 4 | 36 | 16 | 20833 | 1435 | 124 |
|  | 5 | 86 | 98 | 1052 | 30278 | 92 |
|  |  | 130 | 109 | 7094 |  |  |

Table IV. Region Information on P3.

|  | Class |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 |  |
| \# REGIONS $<50$ | 26 | 9 | 44 | 18 | 12 |  |
| TOT. AREA | 319.0 | 190.0 | 603.0 | 354.0 | 228.0 |  |
| AVG. AREA | 12.3 | 21.1 | 13.7 | 19.7 | 19.0 |  |

Table V. Region Information on P 9.

|  |  | Classes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 |
| \# REGIONS | $<50$ | 3 | 3 | 20 | 8 | 1 |
| TOT. AREA |  | 34.0 | 72.0 | 357.0 | 179.0 | 5.0 |
| AVG. AREA |  | 11.3 | 24.0 | 17.9 | 22.4 | 5.0 |

## Conclusions

This research project indicates that a weighted neighborhood approach to class conversion is very promising in processing digital thematic maps. The objectives of a thematic map are met surprisingly well. The main disadvantage is loss of information from the original picture. For example, the islands from the original looses some of its shape and in Figure 3 actually joins the mainland. By manipulating the size of the weighted neighborhood matrix, the weight distribution in the weighted neighborhood matrix, and class weights, a user can "tailor" the boundary smoothing algorithm to suit his needs.

The following specific disadvantages can be cited:

- Pixels near the picture edge utilize only part of the weighted neighborhood matrix and may not be converted to the optimal class.
- A continuous trade-off exists between creating smooth homogeneous regions and distorting the original picture.
- No one definition of a weighted neighborhood matrix is optimal. Users must select a WNHD based on their objectives.
- Every pixel in the picture is examined for conversion causing considerable execution time.


## References

1. Davis, W. A. and Peet, F. G., "The Identification and Reclassification of Small Regions on Digital Thematic Maps", Forest Management Institute Information Report FMT-X-90, September, 1976.


Figure 1. Pl, Original Classified Image.


Figure 2. P2, where NHD $=3$ and whND $=X-S Q R T(z l * * 2+z 2 * * 2)$


Figure 3. P6, where NHD $=5$ and WHND $=\mathrm{X} /(2 * \operatorname{SQRT}(\mathrm{zl} * * 2+z 2 * * 2)+1)$


Figure 4. P9, where NHD $=3$ and WHND $=\mathrm{X} /(2 * \operatorname{SQRT}(\mathrm{z} 1 * * 2+z 2 * * 2)+1)$

