Matching and Intersection of Random Curves

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Intersecting and matching lines is a very important issue in computer mapping, especially for Resource Information Systems where it is used for polygon overlay.

In the early 70's, the senior author has developed a concept for the description and manipulation of cartographic lines that is based on the idea that a line has a thickness. The areal extent of the line increases with decreasing scale. In the same way, the line loses ondulation and length with decreasing scale.

To implement the idea, a bounding rectangle is constructed around a curve - we call it band - with the direction of the band given by the straight line that links its endpoints and its width given by the extreme perpendicular offsets of the line from the straight connector. Each curve can be broken down into subcurves by using the points that touch the boundaries of the band as new endpoints. This process can be continued until a band is identical with a vector and has the width zero. As the number of bands increases, the total area covered by them decreases.

This concept was developed initially as the theoretical explanation of a routine to find salient points along a line (D. Douglas and T.K. Peucker). An initial threshold offset is chosen and the bands are broken down recursively until the offset is less than the threshold offset.

A similar approach was used for the intersection of two lines. Since the points of intersection of two curves must lie within the intersecting parallelogram of their bands, one can discard the portions of the lines that lie outside the parallelogram and recursively construct bands and discard outlying portions until the intersections are found. The implementation of this concept makes it necessary to account for several special cases but also allows for some short-cuts that make the algorithm very fast.

However, this process has two worst cases, one of which - the double line and sliver line - not only converges very slowly but also poses a conceptual problem that needs special treatment. The problem is the fact that most cartographic lines are not precise representations of the feature that they represent but incorporate errors. Therefore, curves are only identifiable within an error band and multiple intersections of curves that create spurious polygons are an indication that the two curves are two representations of one and the same underlying process.

To handle such cases, we developed a line-matching algorithm that incorporates line-intersection as a special case. Given an offset proportional to the estimated error variance, the two lines are banded and the outlying portions dropped. When the width of both bands is less than twice the threshold offset, one band is expanded to that width and the other curve is tested with respect to its position within the band. If the distance between the entrance of the curve into the band and its exit is more than three times the band width (a somewhat arbitrary value), that portion of the two curves is assumed to match and a new curve is constructed that represents a middle line between the two curves. Otherwise the intersection point is found.

The implementation of the concept can be a simple expansion of the intersection algorithm, except that the typical cases (with many curves and re-entrant matches) need a fair amount of bookkeeping.