TOPOLOGY AS A FRAMEWORK FOR SOLID MODELING

(Extended Abstract)

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The idea of using topological information as the framework for a solid modeling representation has essentially been utilized in a number of solid modeling systems ([BAUM72], [BRAI78], [EAST79]), where the sheer amount of space and attention devoted to storing topological relationships becomes a major feature of their approach. The concept itself, however, has not received much explicit treatment in the literature.

The concept merits discussion as a topic in itself, particularly in light of recent information regarding sufficiency of topological representations for solid models. This paper addresses some of the central issues in the use of topological information as a framework for the implementation of solid modeling systems.

Solid Modeling

Solid modeling, first developed in the early 1970's, is an incremental but important improvement over surface modeling techniques developed since the early 1960's for the representation of physical objects for industrial design, analysis, and manufacturing needs.

Solid modeling provides a guarantee that models created form a closed and bounded object more closely related to a physically realizable shape than can be guaranteed for surface models. Solid models, unlike surface models, enable a modeler system to distinguish the outside of a volume from the inside, allowing mass property analysis for the determination of volume, center of gravity, and the like. Typical solid modeling systems also offer tools for the creation and manipulation of such shapes.

A variety of solid modeling representational techniques are available. These techniques can be differentiated on the basis of at least three independent criteria concerning whether the representation is:

— object based or spatially based

— boundary based or volume based

— evaluated or unevaluated in form

Thus many representational techniques are potentially available by choosing different combinations of values of the above criteria. The most appropriate modeling technique to use depends not only on the intended application but also on the particular phase of the application one is concerned with. Many modelers support multiple representational techniques to insure their efficacy over a broad range of applications and phases of the same application.

The application domain of interest to this paper is the design, analysis, and manufacture of solid mechanical parts. Early in the design phase of such objects a high level of abstraction, a symbolic form, offers the most powerful means of performing complex design tasks - as long as the abstraction is appropriate to the design task at hand and the designer performing it. However, during modification, analysis, and use of the constructed model, easy availability of complete information on the model is a prime consideration. For this phase of this application it has been popular to use an object oriented, evaluated, boundary form of solid model. This paper focuses on the use of topological information to provide a framework for the implementation of this particular form of solid modeler.

Topology and Geometry

Geometry is considered here to represent essentially all information about the geometric shape of a solid object including where it lies in space and the precise geometric location of all aspects of its various elements.

Topology, by definition, is an abstraction, a coherent subset, of the information available from the geometry of a shape. More formally, it is a set of properties invariant under a restricted set of geometric transformations. Invariance under transformation implies that all information is not present in topology; topology is incomplete information which can be derived from the complete geometric specification. Topology is not an *arbitrary* subset of geometric information, but rather a carefully selected subset which retains its values under the specified set of manipulations to the object model. Such a coherent subset of information, one that supports a meaningful view of the whole, is the essence of an abstraction.

Given this idea, one can consider topological information as a fuzzy definition of an object located somewhere on the continuum between no information on the object and a complete geometric definition of the object (see Figure 1). As such, topology constrains, but does not uniquely define, the final geometry of an object. On the other hand, a complete geometric description completely defines the topology of an object, though such geometric information may not be in a form convenient for the derivation of topological information.

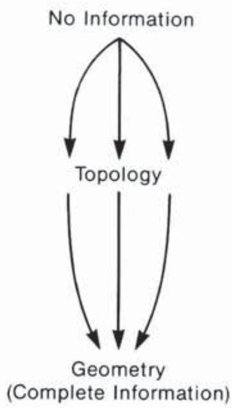

Figure 1. Topological information in the continuum of information about the geometric shape of an object.

Different Kinds of Topology

In the context of solid modeling, when we think of topology we most often think of the adjacencies between topological elements such as vertices, edges, and faces (see Figure 2).

But such adjacency topology is only one subset of many possible subsets of geometric information - only one among many forms of topology. Knot theory topology - knots involving loops in objects which cannot be undone by geometric transformation short of intersecting the objects - is one example of a different form of topology (see Figure 3a). The amount of twist in an object of genus greater than zero is another form of topology which is totally unrelated to adjacency or knot topology. No matter how many complete full turn twists are put into an object, its adjacency relationships remain the same, though the object itself is clearly different (see Figure 3b). In this case all three forms of topology are orthogonal; that is, each has information which is independent of the other two.

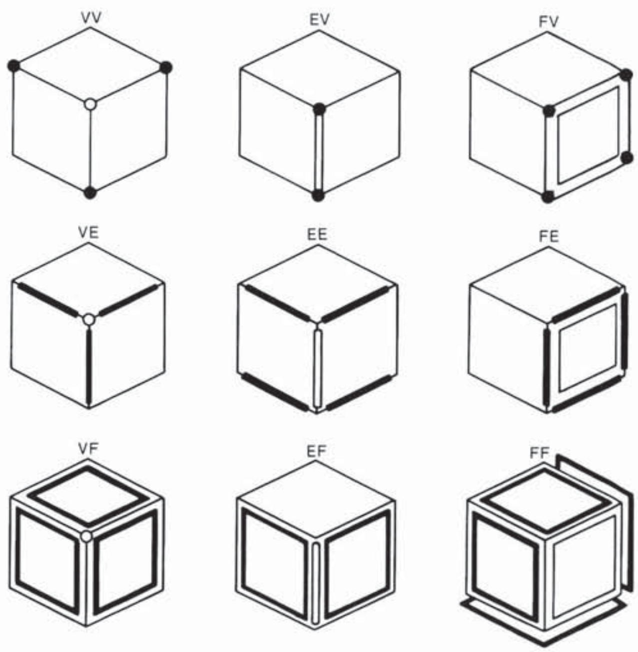

Figure 2. The nine element adjacency relationships in an adjacency topology of faces, edges, and verticies.

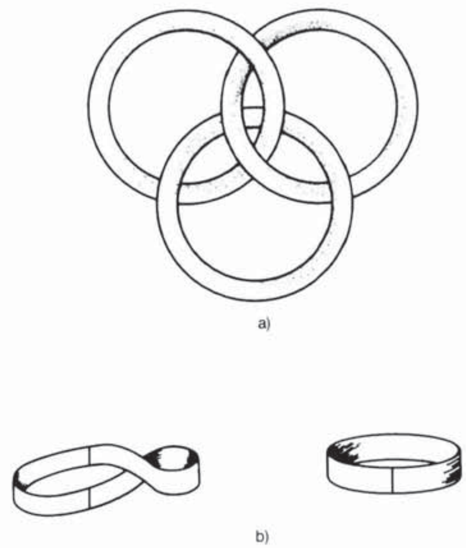

Figure 3. Events of interest in (a) knot topology and (b) twist topolgy.

We will restrict our consideration here to the adjacency form of topology since that form has so far been found the most useful in our selected application area.

Using Topology

What benefit is there to considering the topology of a solid model apart from the complete geometric description?

When it is a unified, coherent high level abstraction of available information, topology is useful in several situations. First, it is useful whenever a concise global abstraction or summary of information can save time over being forced to view in full detail all data associated with a model. Second, during local manipulation of a small portion of an object, it is useful to be able to find adjacent portions of the object without having to review all data associated with the object.

Use of these two properties can simplify manipulation algorithms and greatly improve their efficiency. However, topology can be even more useful when it serves as a framework around which the solid modeling representation can be built.

Topology as a Framework

By using topology as a framework for a solid modeling representation we mean first that topological information is explicitly available and second that it serves as the organizing factor in the schema of the data structures used in the representation (and therefore in the algorithms which operate on the structures). Third, to provide a unified total structure, all topological information must be associated together. To date, the most commonly useful approach has been to organize the topological information in a top-down hierarchical fashion from higher to lower levels of dimensionality (see Figure 4).

The usefulness of topological information as described in the previous section is not the only reason topology should be considered as a framework around which a solid modeling representation can be built. There are more compelling reasons.

First, once the topological and geometric domain which the representation is intended to cover has been defined, and the corresponding topological representation has been selected, the topological portion of the representation remains relatively stable. Geometric surface representation techniques are still a subject of research; the modeling field has not yet converged on any single "ultimate" geometric surface representation technique. As a result, many different forms of geometric surface representation techniques currently exist, and more are under development. If a topological framework is used, old geometric representations can be pulled out and new ones plugged in or multiple geometric representations can be handled simultaneously without major changes to the structure of the system. With a stable topological framework the impact of such geometric representation changes can be minimized to small portions of the system

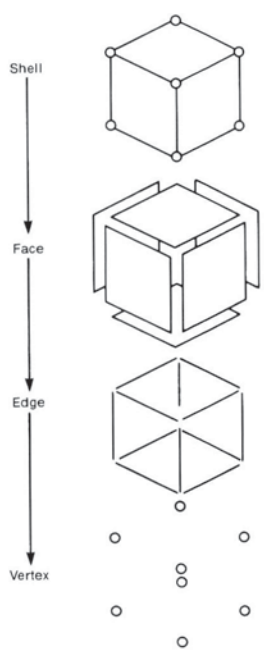

Figure 4. Top-down hierarchy of topological elements from higher to lower levels of dimensionality.

and the ability to add new or replace existing geometric representations is enhanced.

Second, because of the approximate nature of geometric representations of generalized curved surfaces as currently implemented on computers, it is possible in many geometric representations for small gaps to develop between patches that were intended to be adjacent. Relying on geometric information alone to determine topological relationships such as patch to patch adjacency is a risky proposition. In this case, combining a topological framework with one or more geometric surface representation techniques provides a way to represent the properties of an intended object in spite of possible geometric inaccuracies.

Third, separation of topological and geometric information in a solid modeling representation provides a more systematic approach to implementation, providing for simpler creation, verification, and analysis of the model.

Sufficient Topology

In an adjacency topology consisting of three primitive elements such as faces, edges, and vertices, there are nine possible adjacency relationships (as seen in Figure 2). If a topological representation contains enough information to recreate all nine of these adjacency relationships without error, it can be considered a *sufficient* adjacency topology representation.

A complete characterization of a sufficient representation cannot be made without first identifying the domain over which the representation is intended to be valid. A more detailed discussion of domain and sufficiency of topological representations for solid models can be found in [WEIL83].

Since it is not necessary to store all nine adjacency relationships in order to have a sufficient topological representation, identifying a sufficient minimal subset of that information becomes an issue. Three of the nine individual adjacency relationships can be considered sufficient, but topological representations useful in solid modeling normally utilize one of the individually sufficient adjacency relationships in combination with one or more other adjacency relationships [WEIL83]. If the combination selected is sufficient, it is not necessary to rely on geometric information to obtain all remaining topological adjacency relationships. Because of possible inaccuracies in geometric data, a sufficient topological representation is therefore highly desirable.

Sufficient Topology as a Framework

When topological information is used as a framework for solid modeling representations its advantages are best realized if it is independent of geometric representations. Otherwise changes cannot be made to the geometric representation portion of the system without putting the entire framework at risk. In other words, a topological representation chosen as a framework for a solid modeling system should be a sufficient topological representation. The use of a sufficient topological representation for the framework also allows a more complete consistency check against geometry, often avoiding or identifying inconsistencies due to geometric inaccuracy. Furthermore, one can help avoid the inadvertent assumption of sufficient information by algorithms which manipulate the representation.

Conclusion

In an object oriented evaluated boundary form of solid modeler it is highly desirable to utilize an adjacency topology data structure as a framework in the structure of the implementation. The abstraction implicit in this topology-based organization of the data can increase the efficiency and simplicity of the modeling system. For this scheme to gain full advantage, however, the topological information used as the framework must be mathematically sufficient information, independent of the geometric information in the model. In this case the use of topology as a stable framework for the implementation structure can minimize the impact of changes in the geometric representation portions of the system, can help surmount some geometric accuracy problems, and can simplify creation, verification, and analysis of the solid model.

References

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