KINEMATIC AND GEOMETRIC MODELLING AND ANIMATION OF ROBOTS

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ABSTRACT

At present, most industrial robots are programmed in a teach mode. In the meantime, robots are called upon to perform increasingly complex tasks, which makes programming by teaching rather tedious and cumbersome. There is an increasing need for effective tools to assist in off-line programming of robots and verifying their programmed moves.

A program for modelling and simulating the PUMA 560 and ADEPT I robots has been developed. The kinematic models required for the transformation from task space to robot configuration space, and vice-versa, are briefly outlined. The robot geometric models and their graphical manipulation has been implemented using MOVIE·BYU and PLOT-10 on a Tektronix 4115-B graphics terminal and a VAX 11/730 minicomputer. The developed kinematic and geometric models are used for off-line simulation, animation and verification of robot movements during assembly operations. Sample outputs which illustrate the program capabilities such as shaded images, wire-frame animation, arbitrary point tracing, actual movement envelope and the various menus are included.

KEYWORDS: geometric modelling, robot kinematics, graphical simulation and animation, robot off-line programming.

INTRODUCTION

Robots have recently been gaining popularity and acceptance as an efficient tool for accomplishing various manufacturing tasks. Current applications include material handling, welding, painting, deburring, etc. Robots' greatest potential for increasing productivity is in the field of automated and flexible assembly. The key to their success in this area will be in integrating robots with adequate sensors and providing high level sophisticated programming tools.

One of the major goals of our current research in the Centre for Flexible Manufacturing Research and Development at McMaster University is to develop systems which can automatically synthesize programs for controlling robots performing assembly tasks with sensor feedback. The input to these systems includes:

a) specification of assembly tasks and goals, and
b) specification of the initial state of both the robot and world models.

An expert system written in COMMON LISP, currently under development, then uses the knowledge base and production rules to produce a plan for robot motions, and a robot level program in VAL II. The on-line expert system will allow updating of preplanned robot moves in response to sensor input.

Achieving this goal requires several building blocks, including:

1) Robot geometric and kinematic models,
2) Robot world geometric model, including parts and tasks,
3) Sensor models,
4) Motion planner,
5) Adaptive learning model, and
6) Knowledge base and inference engine.

Several research projects are in progress to develop these modules. This paper focusses on the generation of the kinematic and geometric models of the two robots used in the centre, and the links between these models and the rest of the modules mentioned above.

The flexible robot assembly system, installed in the Centre for FMS research and development, consists of two robot work stations, an IRI vision inspection station, a load/unload station, and a computer-controlled Bosch conveyor, with pallets, for material handling. The two robots are a six-axis articulated PUMA 560 and a four-axis ADEPT I scara robot. Both robots are interfaced with force, tactile and vision sensors for real-time adaptive control. The flexible assembly system is used for both research and development projects related to mechanical and electronic assembly.

THE KINEMATIC MODELS

Controlling and programming robots requires analytical models which relate the robot configuration space, expressed in terms of joint variables, and task space normally described in Cartesian coordinates. These models should allow efficient transformation between the two spaces since robots are usually controlled in the joint space while tasks are normally defined in the Cartesian space.

A robot kinematic model consists of two parts:

1) Forward Kinematics: which accomplishes the transformation from configuration space to task space, and
2) Inverse Kinematics: which transforms representations in task space to configuration space.

Kinematic models may be used to transform positions, velocities and accelerations between the two spaces.

Several methods have been developed in recent years to obtain generalized and efficient solutions to these kinematic models [1-8]. The emphasis has been twofold: a) concise and elegant model representation, and b) fast and efficient solutions to these models. The method and notation used in developing our models is based on those developed by Denavit and Hartenburg [1] as applied to robots by Paul [4]. The results are summarized below.

PUMA 560 ARTICULATED ROBOT

PUMA 560 is a 6-degrees-of-freedom articulated robot where the first three joints are used to place the manipulator wrist in its work envelope and the last three joints are used to orients the wrist. All robot joints are rotational.

The coordinate frames used to derive the kinematic models of the PUMA 560 robot are shown in figure 1. The kinematic parameters are summarized in Table 1.

Table 1: PUMA 560 Configuration Constants

Joint #	Joint Variable	Joint Angles Range (degrees) θ_i min.	θ_i max.
1	θ_1	$-160°$	$+160°$
2	θ_2	$-\pi-43°$	$+43°$
3	θ_3	$-52°$	$+\pi+52°$
4	θ_4	$-110°$	$+170°$
5	θ_5	$-100°$	$+100°$
6	θ_6	$-266°$	$+266°$

Joint #	a_i(mm)	d_i (mm)	α_i (degrees)
1	0	0	−90
2	431.81	149.09	0
3	−20.31	0	90
4	0	433.07	−90
5	0	0	90
6	0	56.25	0

The reader is referred to [9] for details of the development of both forward and inverse kinematic model solutions of PUMA type robots. This method was closely followed in developing our PUMA 560 kinematic models.

ADEPT I SCARA ROBOT

ADEPT I robot is a 4-degrees-of-freedom manipulator with two rotational joints (#1 and #2) and one rotational and prismatic joint (#3). Figure 2 shows the kinematic model of the ADEPT I and its link and joint parameters. Table 2 lists the arm kinematic parameters.

Table 2: ADEPT I Configuration Constants

Joint # (i)	Joint Variable	Variable Limits min.	max.	α_i	a_i (mm)	d_i (mm)
1	θ_1	$-150°$	$+150°$	0	425	870
2	θ_2	$-147°$	$+147°$	0	375	223.5
3	θ_3	$-277°$	$+277°$			
	d_3	0	203 mm	0	0	0

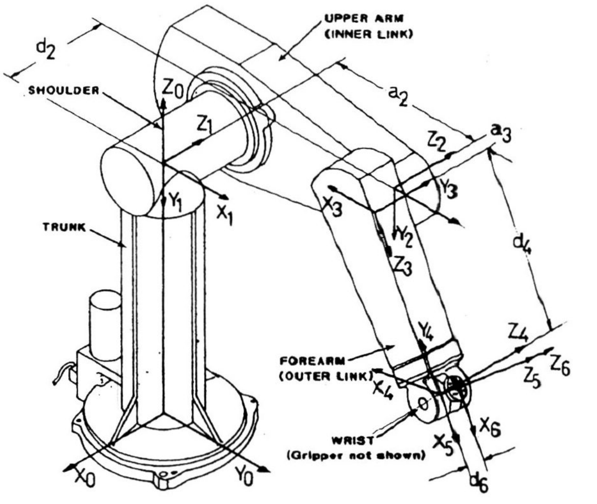

Fig. 1 PUMA 560 coordinate frames and kinematic parameters.

Inverse Kinematics

Using Paul's method, a solution for the inverse kinematics model of ADEPT I was derived.

$$\theta_1 = \tan^{-1} \frac{P_y - P_x\left[(\pm)\sqrt{\frac{1}{A^2} - 1}\right]}{P_y\left[(\pm)\sqrt{\frac{1}{A^2} - 1}\right] + P_x} \tag{1}$$

where

$$A = \frac{P_x^2 + P_y^2 + a_1^2 - a_2^2}{2\,a_1 r} \tag{2}$$

$$r = \sqrt{P_x^2 + P_y^2} \tag{3}$$

and the (+)ve and (−)ve signs in equation (1) refer to the right and left hand robot configurations respectively.

$$\theta_2 = \tan^{-1}\left[\frac{P_y \cos\theta_1 - P_x \sin\theta_1}{P_y \sin\theta_1 + P_x \cos\theta_1 - a_1}\right] \tag{4}$$

$$\theta_3 = 2\tan^{-1}\left[\frac{n_y \cos(\theta_1+\theta_2) - n_x \sin(\theta_1+\theta_2)}{n_y \sin(\theta_1+\theta_2) + n_x \cos(\theta_1+\theta_2)}\right] \tag{5}$$

where

$$n_x = \cos\theta_3(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + \sin\theta_3(-\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2) \tag{6}$$

$$n_y = \cos\theta_3(\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2) + \sin\theta_3(-\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2) \tag{7}$$

$$d_3 = d_1 + d_2 - P_z \tag{8}$$

where the actual linear displacement of joint 3 is equal to $d_3 - d_3'$ and d_3' is a physical offset ($d_3' = 215$ mm).

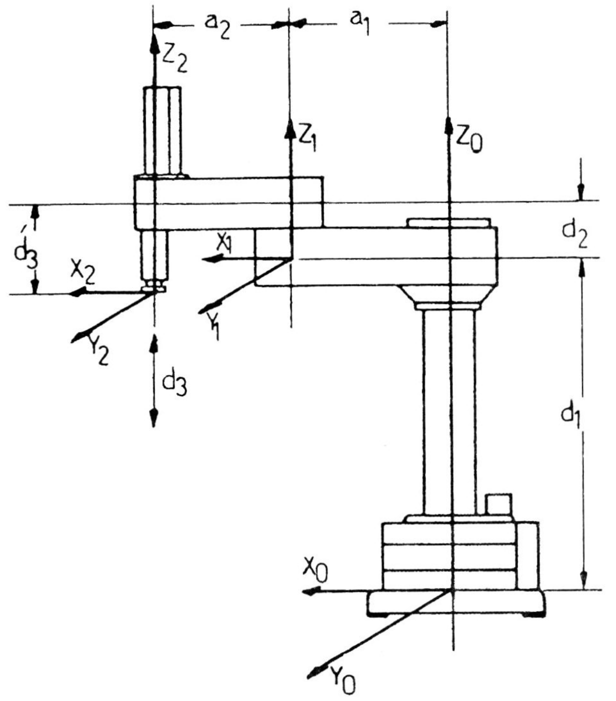

Fig. 2 ADEPT I coordinate frames and kinematic parameters.

Forward Kinematics

The forward kinematics solution defines the Cartesian coordinates of the robot end effector in terms of the robot joint variables as follows:

T_3 = orientation matrix

$$= \begin{vmatrix} n_x & 0_x & a_x \\ n_y & 0_y & a_y \\ n_z & 0_z & a_z \end{vmatrix} \qquad (9)$$

where

$$n_x = \cos(\theta_1 + \theta_2 + \theta_3)$$
$$n_y = \sin(\theta_1 + \theta_2 + \theta_3) \qquad (10)$$
$$n_z = 0$$

$$0_x = \sin(\theta_1 + \theta_2 + \theta_3)$$
$$0_y = -\cos(\theta_1 + \theta_2 + \theta_3) \qquad (11)$$
$$0_z = 0$$

$$a_x = 0\,, \qquad a_y = 0\,, \qquad a_z = -1. \qquad (12)$$

The Cartesian coordinates are given by:

$$P_x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) \qquad (13)$$

$$P_y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2) \qquad (14)$$

$$P_z = d_1 + d_2 - d_3 \qquad (15)$$

The closed form solutions of the forward and inverse kinematic models of the PUMA 560 and ADEPT I were implemented in FORTRAN 77 and linked with a robot geometric modeller and simulator called ROBOT. The Cartesian and Joint Coordinates, calculated using the developed models, were compared with those measured using the two robots in all eight quadrants. The maximum absolute errors were in the range of 0.0028% and 0.06%.

GEOMETRIC MODELLING

Robot motions are not easy to visualize or verify using the mathematical models only. Interactive graphical simulation of robot motion is a powerful tool for verifying roobt programs off-line and evaluating the robot interaction with surrounding objects. It provides for rapid error checking and assists in laying out the robot work cell.

The geometric modelling system MOVIE·BYU was used to generate robots and world models. Wire frames were generated from the geometry data base and used for robot animation to increase display speed. This procedure and the user's menus were developed using GKS PLOT 10 and implemented on a VAX 11/730 minicomputer and a Tektronix 4115B colour display. This approach significantly reduces the overhead normally required by MOVIE·BYU, and maintains the rich details of the graphical model while producing relatively fast images for dynamic display.

PROGRAM OVERVIEW

ROBOT is an interactive program which performs graphical simulation of the kinematics of arbitrary open chain linkages such as robots. ROBOT is primarily menu-driven. All functions are selected from menus displayed on the right hand side of the screen. Keyboard input is only required when parameter values or file names are needed within various functions. Menu selection is done via keyboard, thumbwheels and/or graphics tablet. ROBOT utilizes geometric files created using MOVIE·BYU and kinematic configuration files corresponding to each robot. Currently both PUMA 560 and ADEPT I have been modelled geometrically and kinematically. The program performs the following functions:

1) Reads geometric and kinematic data files corresponding to the selected robot.
2) Allows the user to create or update the kinematic parameters of the robot.
3) Allows the user to specify the geometry and dimensions of end effectors and grippers.
4) Accepts the Cartesian coordinates of the robot end effector, solves the inverse kinematic problem, produces the corresponding joint coordinates and displays the resulting robot configuration.
5) Accepts joint coordinates, solves the forward kinematics problem, produces the Cartesian coordinates and displays the corresponding robot configuration. Joint parameters may be entered as absolute or relative values.
6) Moves the robot to a new configuration either instantly or in a sequence of successive configurations for animation. The number of steps provided by the user is used to create equal step sizes for each joint. Frames may either be left on the screen, resulting in a superimposed sequence of images, or cleared before the next one appears.
7) Allows any point(s) in the geometry to be distinguished and used to trace a path(s) in space during robot animation. This feature is useful in

verifying the path traced by a tool or end effector and in visually checking the collision of the selected part of the robot arm with other objects in the workspace. Vertical walls from the selected point trace may also be viewed to further clarify the 3-D path and work envelope.

8) Allows the user to alter the view of the world. The 3-D graphical representation produces perspective or parallel projection according to the user's selection. The user's point of view is altered by rotating the robot coordinate system (about x, y or z axes) and centering the image on the display screen.
9) Displays in the upper right hand corner of the screen all selected options in the form of ON and OFF switches which can easily be toggled to change the corresponding status.
10) Continuously displays and updates the value of robot joint variables and highlights those violated by a given robot move.
11) Produces stereo pair for hard copy by generating 2 images of the current view. The first image is a perspective projection from the left eye point of view and the second is from the right eye point of view, which is reflected about the vertical axis in the projection plane. When both images are made into hard copies, they result in a stereo image when viewed simultaneously with the aid of a small plain mirror.

Sample outputs are included to illustrate the various options and capabilities of the ROBOT program.

ROBOT GEÓMETRIC MODEL BUILDING

The ROBOT program is a general purpose simulation and animation program capable of simulating any open chain type mechanism. The type of robot, its geometry, kinematics and physical constraints are defined in separate files. Therefore, a library of different robot models may be created for future use. Currently, only the PUMA 560 and ADEPT I robots have been completely modelled in our system.

Each robot is represented as an assembly of robot links. Each link is constructed as a separate object using MOVIE-BYU. The reference coordinate frame for each link is positioned at the link joint according to the Denavit and Hartenburg convention. Therefore, the robot shape and dimensions are modelled as a collection of polygonal surfaces or lines in 3-D space. The geometry consists of Cartesian coordinates in $R \times R \times R$ (called points or nodes) along with a connectivity relation on the nodes which is used to determine which lines are to be drawn in a wire frame model. Any redundant lines produced by the connectivity relation will be deleted and removed by the optimizing module in ROBOT as soon as the robot geometry file has been read. Robot geometry generated in this fashion is stored in files and accessed later by ROBOT. Homogeneous transformation matrices describing the relative translation and rotation between adjacent link coordinate frames are used to transform the modelled links into a complete robot configuration [10].

CONCLUSIONS

The developed kinematic and geometric models of the two robots used in our research program, namely PUMA 560 and ADEPT I, were described. The interactive graphics simulation program, ROBOT, proved very useful in displaying and verifying robot moves. This program will be integrated with a robot task planner which is currently being developed at McMaster University. It will certainly be very useful in checking out the robot programs produced by the robot tasks planner based on geometric and functional reasoning to achieve the stated goals. This planner is being developed in common LISP and the main application domain is mechanical assembly. Other modules are being added to ROBOT to model objects in the workspace.

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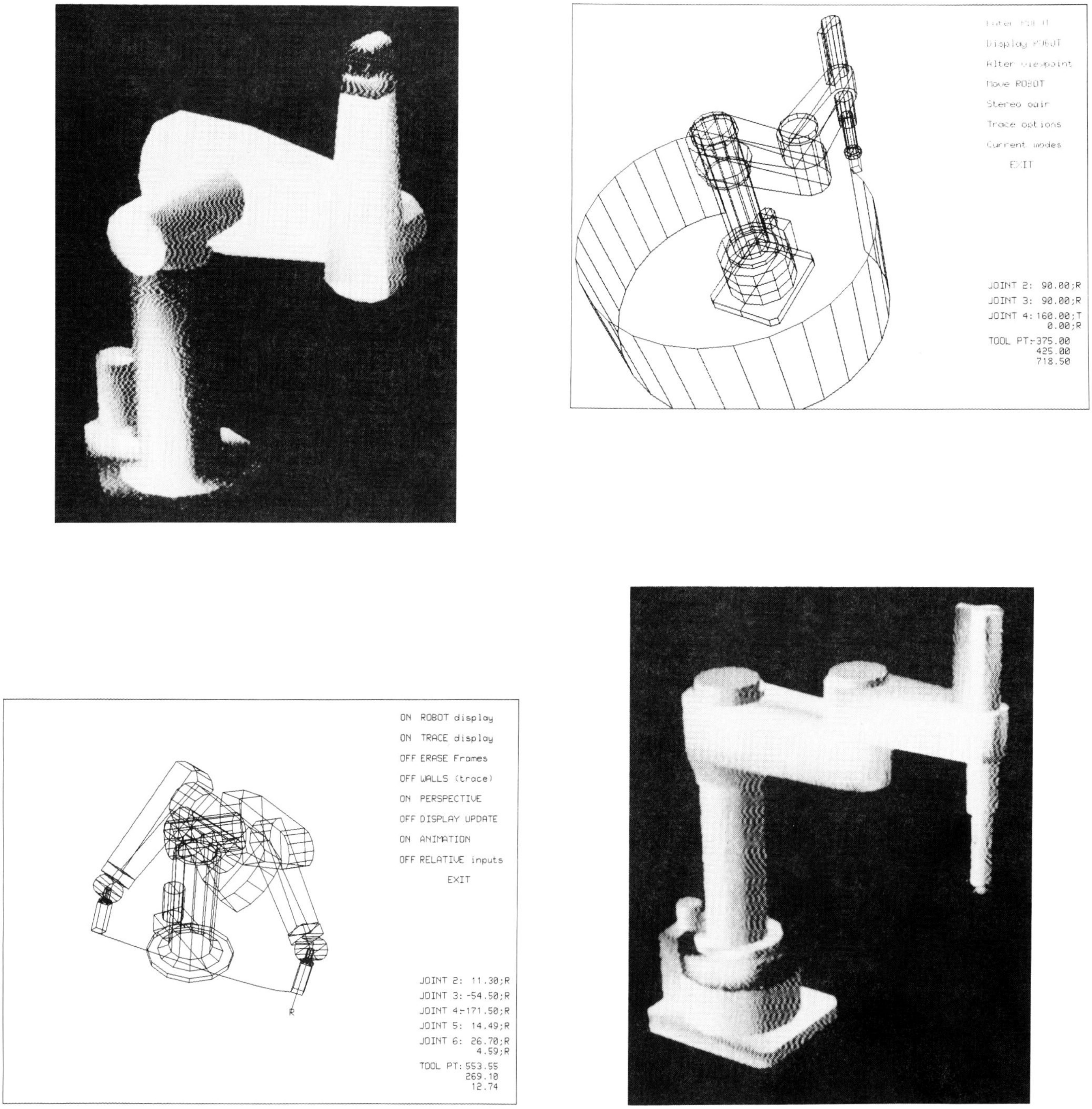

```
JOINT 2:  90.00;R
JOINT 3:  90.00;R
JOINT 4: 160.00;T
           0.00;R
TOOL PT:-375.00
         425.00
         718.50
```

```
ON  ROBOT display
ON  TRACE display
OFF ERASE Frames
OFF WALLS (trace)
ON  PERSPECTIVE
OFF DISPLAY UPDATE
ON  ANIMATION
OFF RELATIVE inputs
        EXIT
```

```
JOINT 2:  11.30;R
JOINT 3: -54.50;R
JOINT 4:-171.50;R
JOINT 5:  14.49;R
JOINT 6:  26.70;R
           4.59;R
TOOL PT: 553.55
         269.10
          12.74
```

Fig. 3 Examples of graphical outputs produced by ROBOT. Actual displays are in colour